ALGEBRAIC GEOMETRY: WORKSHOP 1

In class, we showed that $k[x_1, \ldots, x_n]$ is Noetherian. In the first two problems, we will generalise this a little.

(1) Show that if R is Noetherian and $I \subset R$ is an ideal, then R/I is Noetherian.

(2) Recall the notion of a finitely generated k algebra. Show that every finitely generated k algebra is Noetherian.

In the next few problems, we play with an algebraic set and its defining ideal(s).

(3) Let $X \subset \mathbb{A}^3$ be the union of the coordinate axes. Recall that we proved that (a) finite unions of Zariski closed sets are Zariski closed. Since each coordinate axis is Zariski closed (why?), so is X. Our proof of (a) constructed a set S such that $X = V(S)$. Recall our construction and write down this S.

(4) Let $I = \langle S \rangle$. Is I radical?

(5) Find $I(X)$. Do you see that $I(X) = \sqrt{I}$?