Exercises -

1) Show that a smooth surface in \( \mathbb{P}^3 \) of degree \( \geq 3 \) contains finitely many lines.

2) Let \( \varphi : \mathbb{P}^1 \to \mathbb{P}^n \) be any morphism. Show that the scheme \( \text{Mor}(\mathbb{P}^1, \mathbb{P}^n) \) is smooth at \( \varphi \).

More generally, let \( X \) be any smooth projective variety whose tangent bundle is generated by global sections. Show that \( \text{Mor}(\mathbb{P}^1, X) \) is smooth.

(All "homogeneous varieties" \( X \), like the Grassmannian or flag varieties, have this property.)

3) A famous conjecture due to Clemens states that a general quintic hypersurface \( X \subset \mathbb{P}^4 \) contains finitely many rational curves \( \gamma \) of a given degree. Justify this expectation by showing that the appropriate moduli space has expected \( \dim = 0 \).

4) Consider the Fermat quintic threefold \( C \subset \mathbb{P}^4 \) given by
\[
T = \{ x^5 + y^5 + z^5 + v^5 + w^5 = 0 \}
\]

We can write many lines on \( C \). For example, \( z=v=w=0; \ x=-5_5y; \ z=-5_5v \), where \( 5_5 \) is a fifth root of unity. What is the tangent space of \( \text{Hilb}_T \) at such a line?