Moduli of Curves - Week of Sept 21

Exercises -

1) Show that a smooth surface in $\mathbb{P}^3$ of degree ≥ 3 contains finitely many lines.

2) Let $\varphi : \mathbb{P}^1 \to \mathbb{P}^n$ be any morphism. Show that the scheme $\mathrm{Mor}(\mathbb{P}^1, \mathbb{P}^n)$ is smooth at φ.

   More generally, let X be any smooth projective variety whose tangent bundle is generated by global sections. Show that $\mathrm{Mor}(\mathbb{P}^1, X)$ is smooth.

   (All "homogeneous varieties" X, like the Grassmannian or flag varieties have this property.)

3) A famous conjecture due to Clemens states that a general quintic hypersurface $X \subset \mathbb{P}^4$ contains finitely many rational curves of a given degree. Justify this expectation by showing that the appropriate moduli space has expected dim $= 0$.

4) Consider the Fermat quintic threefold $\subset \mathbb{P}^4$ given by

$$T = \{ X^5 + Y^5 + Z^5 + V^5 + W^5 = 0 \}$$

   We can write many lines on it. For example,

   $\cancel{Z = V = W =}$ $W = 0$; $X = -\zeta_5 Y$, $Z = -\zeta_5 V$, where ζ_5 is a fifth root of unity. What is the tangent space of Hilb_T at such a line?