Calculus I **Final Exam** **21 December 2010**

Name: ____

DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO.

Please put your name on the front page.

This exam is 13 pages long. There are 10 questions for a total of 120 points.

You are not allowed to use the text, your notes, or a calculator on this exam. Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. To receive full credit, you must

- get the correct answer, and
- show your work and/or explain your reasoning that lead to that answer,

unless otherwise noted. Please make sure the solutions you hand in are legible and lucid. You may only use techniques we have developed in class through Section 6.1 of the text.

You will have 2 hours 45 minutes to take this exam. When you are finished with the exam, please return it to the box and initial by your name on the sheet.

Question	Points	Score
1	10	
2	20	
3	15	
4	10	
5	10	
6	10	
7	10	
8	15	
9	10	
10	10	
Total:	120	

1. For the following statements, circle True if the statement is **always** true, and circle False otherwise. Make sure it is completely clear which is your final answer. No explanations are required for this question, and no partial credit. Read the questions very carefully!

   (a) (2 points) If f is a continuous function and $\lim_{z\to 0} f(z) = L$, then $f(0.0001)$ is closer to L than is $f(0.01)$.

   True False

   (b) (2 points) If $f(x)$ and $g(x)$ are twice differentiable functions such that both are concave upwards for all x, then the function $f(x)g(x)$ is also concave upwards for all x.

   True False

   (c) (2 points) If $f'(x)$ is continuous and $f(x)$ has no critical points, then f is everywhere increasing or everywhere decreasing.

   True False

   (d) (2 points) If a is any real number, all antiderivatives of the function $f(x) = x^a$ are multiples of x^{a+1} plus a constant.

   True False

   (e) (2 points) If f is a twice differentiable odd function, then f'' is also odd.

   True False

2. The parts of this question are unrelated to one another, and they are meant to be fairly short.

   (a) (4 points) Find c such that the function $A(x)$ is continuous for all of $\mathbb{R}$:

$$A(x) = \begin{cases} x^2 & \text{for } x < 0 \\ \sin(\frac{\pi}{2}e^x) + c & \text{for } x \geq 0 \end{cases}.$$

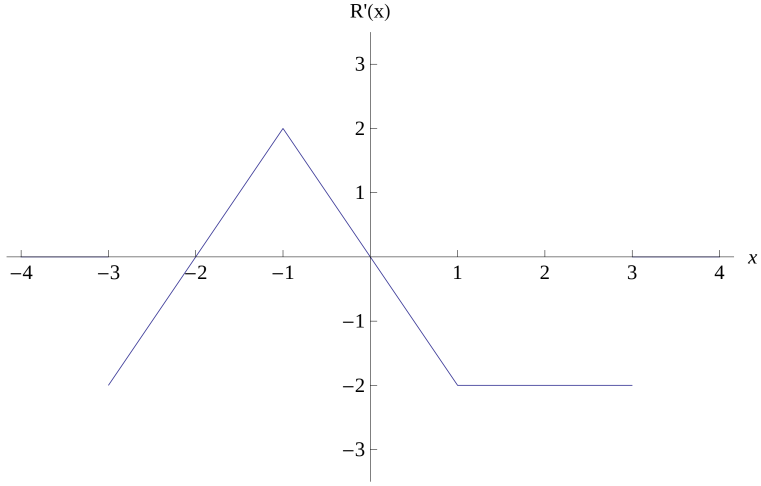
   (b) (4 points) Let $N(x) = x^3 - 2x^2 + 3x + 4$. In using Newton's method to approximate the one real root of N with starting value $x_1 = -1$, what is the next approximation x_2?

   (c) (4 points) If $g(0) = 10$ and $g'(x) \geq 2$ for all x, what is the least possible value for $g(4)$? Justify your answer.

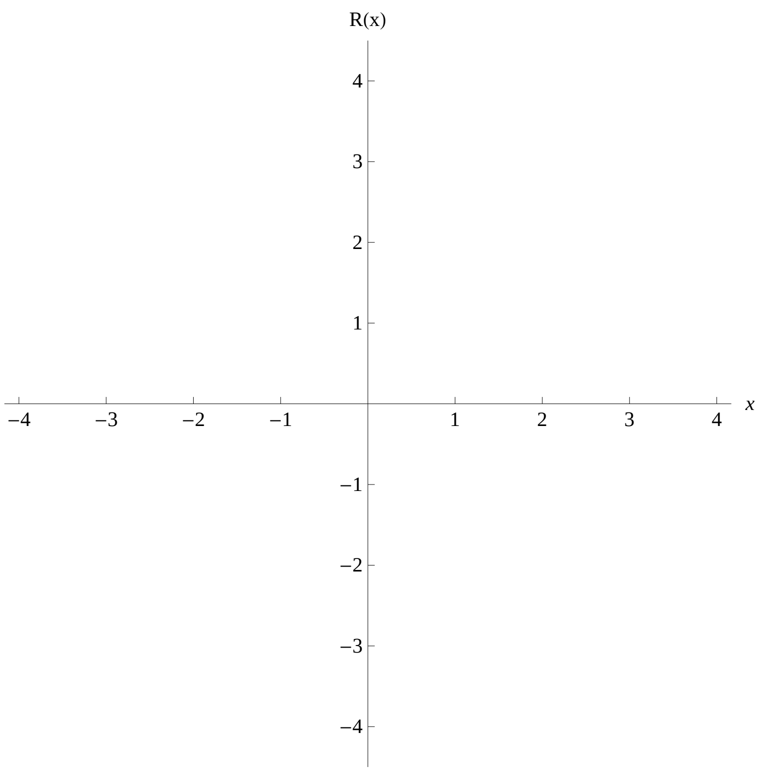
(d) (4 points) If $v(t)$ represents the velocity of a car t minutes after 9:00 am today, explain in words what the value of the integral $\int_{60}^{180} v(t)\,dt$ means.

(e) (4 points) Find the value of $a \neq 1$ such that the average value of the function $h(x) = 2/x^2$ on the interval $[1, a]$ is equal to 1.

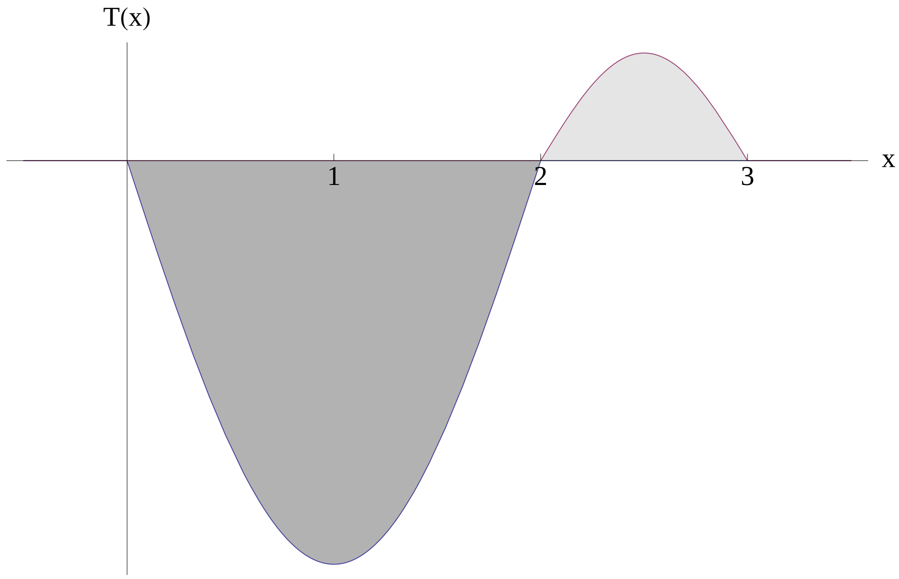
3. (15 points) The graph of the **derivative** of the function $R(x)$ is given below:



If $R(-3) = 1$, sketch the graph of R on the axes below. Give the coordinates of all critical points, inflection points, and endpoints of R on the interval $[-3, 3]$. (Hint: you will need to calculate some of the values of the function R by integrating the derivative.)



4. The following figure shows a portion of the graph of the function $T(x)$, where the region with darker shading has area 15, and the region with lighter shading has area 2.



Suppose the domain of $T(x)$ is all of $\mathbb{R}$, and $T(x)$ is an **even** function. Compute the following:

(a) (2 points) $\int_0^3 T(x)\,dx$

(b) (2 points) $\int_{-2}^{2} |T(x)|\,dx$

(c) (2 points) $\int_{-2}^{3} (T(x)+5)\,dx$

(d) (2 points) $\int_{2}^{-3} T(x)\,dx$

(e) (2 points) $\int_0^{1.5} T(x)\,dx + \int_{1.5}^{3} (T(x)-4)\,dx$

5. (10 points) Compute the following limit, if it exists:

$$\lim_{x \to \infty} x^{\sin(1/x)}$$

6. Suppose f and g are differentiable functions defined on $\mathbb{R}$ with the following known values:

x	-3	-2	-1	0	1	2	3
$f(x)$	1	3	2	-1	-3	-2	0
$f'(x)$	3	2	-2	-3	-1	1	2
$g(x)$	2	3	1	2	4	3	1
$g'(x)$	1	-2	2	3	-1	0	-3

(a) (2 points) If $h(x) = f(g(2x-1))$, compute $h(2)$.

(b) (4 points) If $k(x) = \sin^2(3f(x))$, compute $k'(2)$.

(c) (4 points) Compute $\displaystyle\int_{-3}^{1} \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}\,dx$, assuming $g(x) \neq 0$.

7. (10 points) Suppose that the **derivative** of a function $S(z)$ is given by the formula

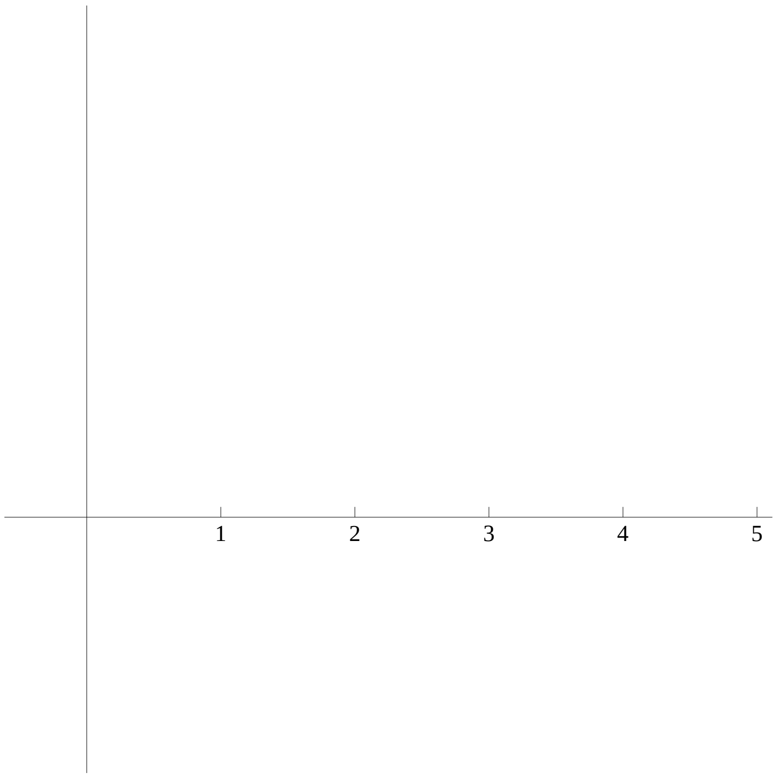
$$S'(z) = \frac{e^{\sin z}(z-5)(z+1000)^{2/3}}{3z+120}.$$

Find all values of z that are critical points of the function S, and determine whether each of these is a local minimum, local maximum, or neither. As always, clearly justify your conclusions.

8. (a) (8 points) Using 6 equal subdivisions of the interval $[1, 4]$, find a Riemann sum that is an **underestimate** for

$$\int_1^4 \ln(2x)\,dx$$

and sketch a graphical representation of your Riemann sum below. There is no need to simplify the sum, and you may either write it out term-by-term or use $\sum$ notation.



(b) (3 points) Show that $\int \ln(2x)\,dx = x\ln(2x) - x + C$, where C is a constant.

(c) (4 points) Use the previous part to find the exact value of the integral $\int_1^4 \ln(2x)\,dx$. Simplify your answer as much as possible.

9. (10 points) Evaluate the integral $\displaystyle\int_0^1 \left(\frac{x^3}{(2x^4+1)^2} + \sec^2\left(\frac{\pi}{4}x\right) \right) dx.$

10. (10 points) Find the shaded area between the curves $y = x^2$ and $y = 6 - x$. You will need to determine where the curves intersect.