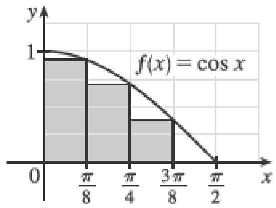
3. (a) $R_4 = \sum_{i=1}^{4} f(x_i)\,\Delta x \quad \left[\Delta x = \dfrac{\pi/2 - 0}{4} = \dfrac{\pi}{8}\right] \quad = \left[\sum_{i=1}^{4} f(x_i)\right]\Delta x$

$= [f(x_1) + f(x_2) + f(x_3) + f(x_4)]\,\Delta x$

$= \left[\cos\frac{\pi}{8} + \cos\frac{2\pi}{8} + \cos\frac{3\pi}{8} + \cos\frac{4\pi}{8}\right]\frac{\pi}{8}$

$\approx (0.9239 + 0.7071 + 0.3827 + 0)\frac{\pi}{8} \approx 0.7908$



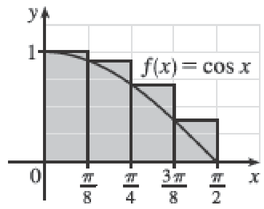
Since f is *decreasing* on $[0, \pi/2]$, an *underestimate* is obtained by using the *right* endpoint approximation, R_4.

(b) $L_4 = \sum_{i=1}^{4} f(x_{i-1})\,\Delta x = \left[\sum_{i=1}^{4} f(x_{i-1})\right]\Delta x$

$= [f(x_0) + f(x_1) + f(x_2) + f(x_3)]\,\Delta x$

$= \left[\cos 0 + \cos\frac{\pi}{8} + \cos\frac{2\pi}{8} + \cos\frac{3\pi}{8}\right]\frac{\pi}{8}$

$\approx (1 + 0.9239 + 0.7071 + 0.3827)\frac{\pi}{8} \approx 1.1835$



L_4 is an overestimate. Alternatively, we could just add the area of the leftmost upper rectangle and subtract the area of the rightmost lower rectangle; that is, $L_4 = R_4 + f(0)\cdot\frac{\pi}{8} - f\left(\frac{\pi}{2}\right)\cdot\frac{\pi}{8}$.

18. For an increasing function, using left endpoints gives us an underestimate and using right endpoints results in an overestimate. We will use M_6 to get an estimate. $\Delta t = \frac{30-0}{6} = 5 \text{ s} = \frac{5}{3600} \text{ h} = \frac{1}{720} \text{ h}$.

$$M_6 = \tfrac{1}{720}[v(2.5) + v(7.5) + v(12.5) + v(17.5) + v(22.5) + v(27.5)]$$
$$= \tfrac{1}{720}(31.25 + 66 + 88 + 103.5 + 113.75 + 119.25) = \tfrac{1}{720}(521.75) \approx 0.725 \text{ km}$$

For a very rough check on the above calculation, we can draw a line from $(0,0)$ to $(30,120)$ and calculate the area of the triangle: $\frac{1}{2}(30)(120) = 1800$. Divide by 3600 to get 0.5, which is clearly an underestimate, making our midpoint estimate of 0.725 seem reasonable. Of course, answers will vary due to different readings of the graph.

22. $\lim\limits_{n\to\infty} \sum_{i=1}^{n} \dfrac{2}{n}\left(5 + \dfrac{2i}{n}\right)^{10}$ can be interpreted as the area of the region lying under the graph of $y = (5+x)^{10}$ on the interval $[0, 2]$, since for $y = (5+x)^{10}$ on $[0,2]$ with $\Delta x = \dfrac{2-0}{n} = \dfrac{2}{n}$, $x_i = 0 + i\,\Delta x = \dfrac{2i}{n}$, and $x_i^* = x_i$, the expression for the area is $A = \lim\limits_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\,\Delta x = \lim\limits_{n\to\infty}\sum_{i=1}^{n}\left(5 + \dfrac{2i}{n}\right)^{10}\dfrac{2}{n}$. Note that the answer is not unique. We could use $y = x^{10}$ on $[5, 7]$ or, in general, $y = ((5-n)+x)^{10}$ on $[n, n+2]$.

17. On $[2, 6]$, $\lim\limits_{n\to\infty}\sum_{i=1}^{n} x_i \ln(1 + x_i^2)\,\Delta x = \int_2^6 x\ln(1+x^2)\,dx$.

18. On $[\pi, 2\pi]$, $\lim\limits_{n\to\infty}\sum_{i=1}^{n} \dfrac{\cos x_i}{x_i}\,\Delta x = \displaystyle\int_{\pi}^{2\pi} \frac{\cos x}{x}\,dx$.

34. (a) $\int_0^2 g(x)\,dx = \frac{1}{2} \cdot 4 \cdot 2 = 4$ [area of a triangle]

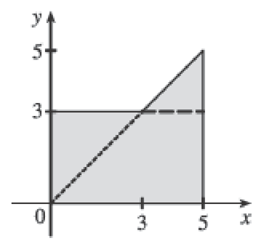
(b) $\int_2^6 g(x)\,dx = -\frac{1}{2}\pi(2)^2 = -2\pi$ [negative of the area of a semicircle]

(c) $\int_6^7 g(x)\,dx = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$ [area of a triangle]

$\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx = 4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi$

48. $\int_1^4 f(x)\,dx = \int_1^5 f(x)\,dx - \int_4^5 f(x)\,dx = 12 - 3.6 = 8.4$

50. If $f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$, then $\int_0^5 f(x)\,dx$ can be interpreted as the area of the shaded region, which consists of a 5-by-3 rectangle surmounted by an isosceles right triangle whose legs have length 2. Thus, $\int_0^5 f(x)\,dx = 5(3) + \frac{1}{2}(2)(2) = 17$.



2. (a) $g(x) = \int_0^x f(t)\,dt$, so $g(0) = \int_0^0 f(t)\,dt = 0$.

$g(1) = \int_0^1 f(t)\,dt = \frac{1}{2} \cdot 1 \cdot 1$ [area of triangle] $= \frac{1}{2}$.

$g(2) = \int_0^2 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt$ [below the x-axis]

$= \frac{1}{2} - \frac{1}{2} \cdot 1 \cdot 1 = 0$.

$g(3) = g(2) + \int_2^3 f(t)\,dt = 0 - \frac{1}{2} \cdot 1 \cdot 1 = -\frac{1}{2}$.

$g(4) = g(3) + \int_3^4 f(t)\,dt = -\frac{1}{2} + \frac{1}{2} \cdot 1 \cdot 1 = 0$.

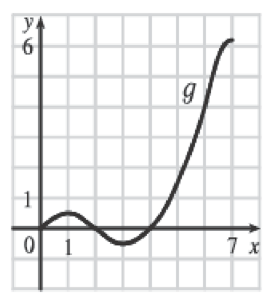
$g(5) = g(4) + \int_4^5 f(t)\,dt = 0 + 1.5 = 1.5$.

$g(6) = g(5) + \int_5^6 f(t)\,dt = 1.5 + 2.5 = 4$.

(b) $g(7) = g(6) + \int_6^7 f(t)\,dt \approx 4 + 2.2$ [estimate from the graph] $= 6.2$.

(c) The answers from part (a) and part (b) indicate that g has a minimum at $x = 3$ and a maximum at $x = 7$. This makes sense from the graph of f since we are subtracting area on $1 < x < 3$ and adding area on $3 < x < 7$.

(d)



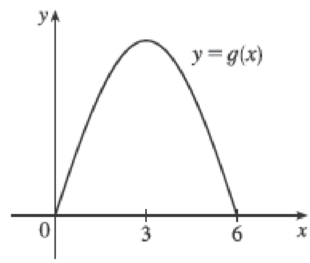
4. (a) $g(x) = \int_0^x f(t)\,dt$, so $g(0) = 0$ since the limits of integration are equal and $g(6) = 0$ since the areas above and below the x-axis are equal.

(b) $g(1)$ is the area under the curve from 0 to 1, which includes two unit squares and about 80% to 90% of a third unit square, so $g(1) \approx 2.8$. Similarly, $g(2) \approx 4.9$ and $g(3) \approx 5.7$. Now $g(3) - g(2) \approx 0.8$, so $g(4) \approx g(3) - 0.8 \approx 4.9$ by the symmetry of f about $x = 3$. Likewise, $g(5) \approx 2.8$.

(c) As we go from $x = 0$ to $x = 3$, we are adding area, so g increases on the interval $(0, 3)$.

(d) g increases on $(0, 3)$ and decreases on $(3, 6)$ [where we are subtracting area], so g has a maximum value at $x = 3$.

(e) A graph of g must have a maximum at $x = 3$, be symmetric about $x = 3$, and have zeros at $x = 0$ and $x = 6$.



(f) If we sketch the graph of g' by estimating slopes on the graph of g (as in Section 2.8), we get a graph that looks like f (as indicated by FTC1).

7. $f(t) = \dfrac{1}{t^3+1}$ and $g(x) = \displaystyle\int_1^x \frac{1}{t^3+1}\,dt$, so by FTC1, $g'(x) = f(x) = \dfrac{1}{x^3+1}$. Note that the lower limit, 1, could be any real number greater than -1 and not affect this answer.

10. $f(x) = \sqrt{x^2+4}$ and $g(r) = \int_0^r \sqrt{x^2+4}\,dx$, so by FTC1, $g'(r) = f(r) = \sqrt{r^2+4}$.

26. $\int_{-5}^{5} e\,dx = \Big[ex\Big]_{-5}^{5} = 5e - (-5e) = 10e$

37. $\displaystyle\int_0^1 (x^e + e^x)\,dx = \left[\frac{x^{e+1}}{e+1} + e^x\right]_0^1 = \left(\frac{1}{e+1} + e\right) - (0+1) = \frac{1}{e+1} + e - 1$

41. $\int_{-1}^{1} e^{u+1}\,du = \left[e^{u+1}\right]_{-1}^{1} = e^2 - e^0 = e^2 - 1$ [or start with $e^{u+1} = e^u e^1$]

61. $y = \displaystyle\int_0^x \frac{t^2}{t^2+t+2}\,dt \quad\Rightarrow\quad y' = \frac{x^2}{x^2+x+2} \quad\Rightarrow$

$$y'' = \frac{(x^2+x+2)(2x) - x^2(2x+1)}{(x^2+x+2)^2} = \frac{2x^3+2x^2+4x-2x^3-x^2}{(x^2+x+2)^2} = \frac{x^2+4x}{(x^2+x+2)^2} = \frac{x(x+4)}{(x^2+x+2)^2}.$$

The curve y is concave downward when $y'' < 0$; that is, on the interval $(-4, 0)$.

78. $B = 3A \quad\Rightarrow\quad \int_0^b e^x\,dx = 3\int_0^a e^x\,dx \quad\Rightarrow\quad [e^x]_0^b = 3\,[e^x]_0^a \quad\Rightarrow\quad e^b - 1 = 3(e^a - 1) \quad\Rightarrow\quad e^b = 3e^a - 2 \quad\Rightarrow$ $b = \ln(3e^a - 2)$