1. (a) $\frac{d}{dx}(9x^2 - y^2) = \frac{d}{dx}(1) \quad \Rightarrow \quad 18x - 2y\,y' = 0 \quad \Rightarrow \quad 2y\,y' = 18x \quad \Rightarrow \quad y' = \frac{9x}{y}$

(b) $9x^2 - y^2 = 1 \quad \Rightarrow \quad y^2 = 9x^2 - 1 \quad \Rightarrow \quad y = \pm\sqrt{9x^2-1}$, so $y' = \pm\frac{1}{2}(9x^2-1)^{-1/2}(18x) = \pm\frac{9x}{\sqrt{9x^2-1}}$.

(c) From part (a), $y' = \frac{9x}{y} = \frac{9x}{\pm\sqrt{9x^2-1}}$, which agrees with part (b).

25. $y\sin 2x = x\cos 2y \quad \Rightarrow \quad y\cdot\cos 2x\cdot 2 + \sin 2x\cdot y' = x(-\sin 2y\cdot 2y') + \cos(2y)\cdot 1 \quad \Rightarrow$

$\sin 2x\cdot y' + 2x\sin 2y\cdot y' = -2y\cos 2x + \cos 2y \quad \Rightarrow$

$y'(\sin 2x + 2x\sin 2y) = -2y\cos 2x + \cos 2y \quad \Rightarrow \quad y' = \frac{-2y\cos 2x + \cos 2y}{\sin 2x + 2x\sin 2y}$. When $x = \frac{\pi}{2}$ and $y = \frac{\pi}{4}$, we have

$y' = \frac{(-\pi/2)(-1) + 0}{0 + \pi\cdot 1} = \frac{\pi/2}{\pi} = \frac{1}{2}$, so an equation of the tangent line is $y - \frac{\pi}{4} = \frac{1}{2}(x - \frac{\pi}{2})$, or $y = \frac{1}{2}x$.

28. $x^2 + 2xy - y^2 + x = 2 \quad \Rightarrow \quad 2x + 2(x\,y' + y\cdot 1) - 2y\,y' + 1 = 0 \quad \Rightarrow \quad 2x\,y' - 2y\,y' = -2x - 2y - 1 \quad \Rightarrow$

$y'(2x - 2y) = -2x - 2y - 1 \quad \Rightarrow \quad y' = \frac{-2x - 2y - 1}{2x - 2y}$. When $x = 1$ and $y = 2$, we have

$y' = \frac{-2 - 4 - 1}{2 - 4} = \frac{-7}{-2} = \frac{7}{2}$, so an equation of the tangent line is $y - 2 = \frac{7}{2}(x - 1)$ or $y = \frac{7}{2}x - \frac{3}{2}$.

32. $y^2(y^2 - 4) = x^2(x^2 - 5) \quad \Rightarrow \quad y^4 - 4y^2 = x^4 - 5x^2 \quad \Rightarrow \quad 4y^3\,y' - 8y\,y' = 4x^3 - 10x$.

When $x = 0$ and $y = -2$, we have $-32y' + 16y' = 0 \quad \Rightarrow \quad -16y' = 0 \quad \Rightarrow \quad y' = 0$, so an equation of the tangent line is $y + 2 = 0(x - 0)$ or $y = -2$.

45. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \Rightarrow \quad \frac{2x}{a^2} - \frac{2yy'}{b^2} = 0 \quad \Rightarrow \quad y' = \frac{b^2x}{a^2y} \quad \Rightarrow$ an equation of the tangent line at (x_0, y_0) is

$y - y_0 = \frac{b^2x_0}{a^2y_0}(x - x_0)$. Multiplying both sides by $\frac{y_0}{b^2}$ gives $\frac{y_0y}{b^2} - \frac{y_0^2}{b^2} = \frac{x_0x}{a^2} - \frac{x_0^2}{a^2}$. Since (x_0, y_0) lies on the hyperbola, we have $\frac{x_0x}{a^2} - \frac{y_0y}{b^2} = \frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1$.

75. $x^2y^2 + xy = 2 \quad \Rightarrow \quad x^2\cdot 2yy' + y^2\cdot 2x + x\cdot y' + y\cdot 1 = 0 \quad \Leftrightarrow \quad y'(2x^2y + x) = -2xy^2 - y \quad \Leftrightarrow$

$y' = -\frac{2xy^2 + y}{2x^2y + x}$. So $-\frac{2xy^2 + y}{2x^2y + x} = -1 \quad \Leftrightarrow \quad 2xy^2 + y = 2x^2y + x \quad \Leftrightarrow \quad y(2xy + 1) = x(2xy + 1) \quad \Leftrightarrow$

$y(2xy + 1) - x(2xy + 1) = 0 \quad \Leftrightarrow \quad (2xy + 1)(y - x) = 0 \quad \Leftrightarrow \quad xy = -\frac{1}{2}$ or $y = x$. But $xy = -\frac{1}{2} \quad \Rightarrow$

$x^2y^2 + xy = \frac{1}{4} - \frac{1}{2} \neq 2$, so we must have $x = y$. Then $x^2y^2 + xy = 2 \quad \Rightarrow \quad x^4 + x^2 = 2 \quad \Leftrightarrow \quad x^4 + x^2 - 2 = 0 \quad \Leftrightarrow$

$(x^2 + 2)(x^2 - 1) = 0$. So $x^2 = -2$, which is impossible, or $x^2 = 1 \quad \Leftrightarrow \quad x = \pm 1$. Since $x = y$, the points on the curve where the tangent line has a slope of -1 are $(-1, -1)$ and $(1, 1)$.

4. $f(x) = \ln(\sin^2 x) = \ln(\sin x)^2 = 2\ln|\sin x| \quad \Rightarrow \quad f'(x) = 2 \cdot \dfrac{1}{\sin x} \cdot \cos x = 2\cot x$

15. $F(s) = \ln\ln s \quad \Rightarrow \quad F'(s) = \dfrac{1}{\ln s}\dfrac{d}{ds}\ln s = \dfrac{1}{\ln s} \cdot \dfrac{1}{s} = \dfrac{1}{s\ln s}$

29. $f(x) = \ln(x^2 - 2x) \quad \Rightarrow \quad f'(x) = \dfrac{1}{x^2 - 2x}(2x - 2) = \dfrac{2(x-1)}{x(x-2)}$.

$\mathrm{Dom}(f) = \{x \mid x(x-2) > 0\} = (-\infty, 0) \cup (2, \infty)$.

42. $y = \sqrt{x}\, e^{x^2 - x}(x+1)^{2/3} \quad \Rightarrow \quad \ln y = \ln\left[x^{1/2}e^{x^2-x}(x+1)^{2/3}\right] \quad \Rightarrow$

$\ln y = \frac{1}{2}\ln x + (x^2 - x) + \frac{2}{3}\ln(x+1) \quad \Rightarrow \quad \dfrac{1}{y}y' = \dfrac{1}{2} \cdot \dfrac{1}{x} + 2x - 1 + \dfrac{2}{3} \cdot \dfrac{1}{x+1} \quad \Rightarrow$

$y' = y\left(\dfrac{1}{2x} + 2x - 1 + \dfrac{2}{3x+3}\right) \quad \Rightarrow \quad y' = \sqrt{x}\, e^{x^2-x}(x+1)^{2/3}\left(\dfrac{1}{2x} + 2x - 1 + \dfrac{2}{3x+3}\right)$

48. $y = (\sin x)^{\ln x} \quad \Rightarrow \quad \ln y = \ln(\sin x)^{\ln x} \quad \Rightarrow \quad \ln y = \ln x \cdot \ln\sin x \quad \Rightarrow \quad \dfrac{1}{y}y' = \ln x \cdot \dfrac{1}{\sin x} \cdot \cos x + \ln\sin x \cdot \dfrac{1}{x} \quad \Rightarrow$

$y' = y\left(\ln x \cdot \dfrac{\cos x}{\sin x} + \dfrac{\ln\sin x}{x}\right) \quad \Rightarrow \quad y' = (\sin x)^{\ln x}\left(\ln x \cot x + \dfrac{\ln\sin x}{x}\right)$

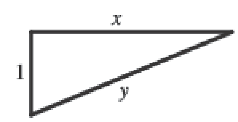
5. $V = \pi r^2 h = \pi(5)^2 h = 25\pi h \quad \Rightarrow \quad \dfrac{dV}{dt} = 25\pi\dfrac{dh}{dt} \quad \Rightarrow \quad 3 = 25\pi\dfrac{dh}{dt} \quad \Rightarrow \quad \dfrac{dh}{dt} = \dfrac{3}{25\pi}$ m/min.

6. $V = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad \dfrac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\dfrac{dr}{dt} \quad \Rightarrow \quad \dfrac{dV}{dt} = 4\pi\left(\frac{1}{2} \cdot 80\right)^2(4) = 25{,}600\pi$ mm^3/s.

11. (a) Given: a plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. If we let t be time (in hours) and x be the horizontal distance traveled by the plane (in mi), then we are given that $dx/dt = 500$ mi/h.

(b) Unknown: the rate at which the distance from the plane to the station is increasing when it is 2 mi from the station. If we let y be the distance from the plane to the station, then we want to find dy/dt when $y = 2$ mi.

(c)



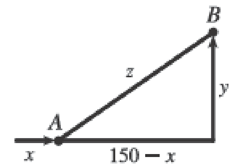
(d) By the Pythagorean Theorem, $y^2 = x^2 + 1 \quad \Rightarrow \quad 2y\,(dy/dt) = 2x\,(dx/dt)$.

(e) $\dfrac{dy}{dt} = \dfrac{x}{y}\dfrac{dx}{dt} = \dfrac{x}{y}(500)$. Since $y^2 = x^2 + 1$, when $y = 2$, $x = \sqrt{3}$, so $\dfrac{dy}{dt} = \dfrac{\sqrt{3}}{2}(500) = 250\sqrt{3} \approx 433$ mi/h.

14. (a) Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km/h, and ship B is sailing north at 25 km/h. If we let t be time (in hours), x be the distance traveled by ship A (in km), and y be the distance traveled by ship B (in km), then we are given that $dx/dt = 35$ km/h and $dy/dt = 25$ km/h.

(b) Unknown: the rate at which the distance between the ships is changing at 4:00 PM. If we let z be the distance between the ships, then we want to find dz/dt when $t = 4$ h.

(c)

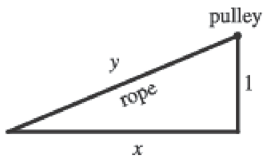


(d) $z^2 = (150 - x)^2 + y^2 \quad \Rightarrow \quad 2z\,\dfrac{dz}{dt} = 2(150 - x)\left(-\dfrac{dx}{dt}\right) + 2y\,\dfrac{dy}{dt}$

(e) At 4:00 PM, $x = 4(35) = 140$ and $y = 4(25) = 100 \quad \Rightarrow \quad z = \sqrt{(150 - 140)^2 + 100^2} = \sqrt{10{,}100}$.

So $\dfrac{dz}{dt} = \dfrac{1}{z}\left[(x - 150)\,\dfrac{dx}{dt} + y\,\dfrac{dy}{dt}\right] = \dfrac{-10(35) + 100(25)}{\sqrt{10{,}100}} = \dfrac{215}{\sqrt{101}} \approx 21.4$ km/h.

20.



Given $\dfrac{dy}{dt} = -1$ m/s, find $\dfrac{dx}{dt}$ when $x = 8$ m. $y^2 = x^2 + 1 \quad \Rightarrow \quad 2y\,\dfrac{dy}{dt} = 2x\,\dfrac{dx}{dt} \quad \Rightarrow$ $\dfrac{dx}{dt} = \dfrac{y}{x}\dfrac{dy}{dt} = -\dfrac{y}{x}$. When $x = 8$, $y = \sqrt{65}$, so $\dfrac{dx}{dt} = -\dfrac{\sqrt{65}}{8}$. Thus, the boat approaches the dock at $\dfrac{\sqrt{65}}{8} \approx 1.01$ m/s.

33. Differentiating both sides of $PV = C$ with respect to t and using the Product Rule gives us $P\,\dfrac{dV}{dt} + V\dfrac{dP}{dt} = 0 \quad \Rightarrow$ $\dfrac{dV}{dt} = -\dfrac{V}{P}\dfrac{dP}{dt}$. When $V = 600$, $P = 150$ and $\dfrac{dP}{dt} = 20$, so we have $\dfrac{dV}{dt} = -\dfrac{600}{150}(20) = -80$. Thus, the volume is decreasing at a rate of 80 cm^3/min.

35. With $R_1 = 80$ and $R_2 = 100$, $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{1}{80} + \dfrac{1}{100} = \dfrac{180}{8000} = \dfrac{9}{400}$, so $R = \dfrac{400}{9}$. Differentiating $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ with respect to t, we have $-\dfrac{1}{R^2}\,\dfrac{dR}{dt} = -\dfrac{1}{R_1^2}\,\dfrac{dR_1}{dt} - \dfrac{1}{R_2^2}\,\dfrac{dR_2}{dt} \quad \Rightarrow \quad \dfrac{dR}{dt} = R^2\left(\dfrac{1}{R_1^2}\,\dfrac{dR_1}{dt} + \dfrac{1}{R_2^2}\,\dfrac{dR_2}{dt}\right)$. When $R_1 = 80$ and $R_2 = 100$, $\dfrac{dR}{dt} = \dfrac{400^2}{9^2}\left[\dfrac{1}{80^2}(0.3) + \dfrac{1}{100^2}(0.2)\right] = \dfrac{107}{810} \approx 0.132\ \Omega/\text{s}$.