

3. $f(x) = 2^{40}$ is a constant function, so its derivative is 0, that is, $f'(x) = 0$.

6. $F(x) = \frac{3}{4}x^8 \Rightarrow F'(x) = \frac{3}{4}(8x^7) = 6x^7$

13. $A(s) = -\frac{12}{s^5} = -12s^{-5} \Rightarrow A'(s) = -12(-5s^{-6}) = 60s^{-6}$ or $60/s^6$

16. $h(t) = \sqrt[4]{t} - 4e^t = t^{1/4} - 4e^t \Rightarrow h'(t) = \frac{1}{4}t^{-3/4} - 4(e^t) = \frac{1}{4}t^{-3/4} - 4e^t$

26. $k(r) = e^r + r^e \Rightarrow k'(r) = e^r + er^{e-1}$

34. $y = x^4 + 2x^2 - x \Rightarrow y' = 4x^3 + 4x - 1$. At $(1, 2)$, $y' = 7$ and an equation of the tangent line is $y - 2 = 7(x - 1)$ or $y = 7x - 5$.

43. $f(x) = 10x^{10} + 5x^5 - x \Rightarrow f'(x) = 100x^9 + 25x^4 - 1 \Rightarrow f''(x) = 900x^8 + 100x^3$

47. (a) $s = t^3 - 3t \Rightarrow v(t) = s'(t) = 3t^2 - 3 \Rightarrow a(t) = v'(t) = 6t$

(b) $a(2) = 6(2) = 12 \text{ m/s}^2$

(c) $v(t) = 3t^2 - 3 = 0$ when $t^2 = 1$, that is, $t = 1$ [$t \geq 0$] and $a(1) = 6 \text{ m/s}^2$.

62. (a) $f(x) = x^n \Rightarrow f'(x) = nx^{n-1} \Rightarrow f''(x) = n(n-1)x^{n-2} \Rightarrow \dots \Rightarrow$

$$f^{(n)}(x) = n(n-1)(n-2)\cdots 2 \cdot 1x^{n-n} = n!$$

(b) $f(x) = x^{-1} \Rightarrow f'(x) = (-1)x^{-2} \Rightarrow f''(x) = (-1)(-2)x^{-3} \Rightarrow \dots \Rightarrow$

$$f^{(n)}(x) = (-1)(-2)(-3)\cdots(-n)x^{-(n+1)} = (-1)^n n! x^{-(n+1)} \text{ or } \frac{(-1)^n n!}{x^{n+1}}$$

77. *Solution 1:* Let $f(x) = x^{1000}$. Then, by the definition of a derivative, $f'(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{x^{1000} - 1}{x - 1}$.

But this is just the limit we want to find, and we know (from the Power Rule) that $f'(x) = 1000x^{999}$, so

$$f'(1) = 1000(1)^{999} = 1000. \text{ So } \lim_{x \rightarrow 1} \frac{x^{1000} - 1}{x - 1} = 1000.$$

Solution 2: Note that $(x^{1000} - 1) = (x - 1)(x^{999} + x^{998} + x^{997} + \cdots + x^2 + x + 1)$. So

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{x^{1000} - 1}{x - 1} &= \lim_{x \rightarrow 1} \frac{(x - 1)(x^{999} + x^{998} + x^{997} + \cdots + x^2 + x + 1)}{x - 1} = \lim_{x \rightarrow 1} (x^{999} + x^{998} + x^{997} + \cdots + x^2 + x + 1) \\ &= \underbrace{1 + 1 + 1 + \cdots + 1 + 1 + 1}_{1000 \text{ ones}} = 1000, \text{ as above.} \end{aligned}$$