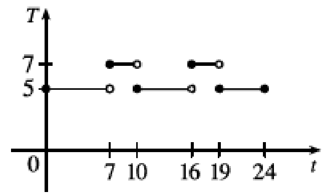
9. (a) The toll is \$7 between 7:00 AM and 10:00 AM and between 4:00 PM and 7:00 PM.

(b) The function T has jump discontinuities at $t = 7, 10, 16$, and 19. Their significance to someone who uses the road is that, because of the sudden jumps in the toll, they may want to avoid the higher rates between $t = 7$ and $t = 10$ and between $t = 16$ and $t = 19$ if feasible.



11. If f and g are continuous and $g(2) = 6$, then $\lim_{x\to 2}[3f(x) + f(x)\,g(x)] = 36 \;\Rightarrow$

$3\lim_{x\to 2} f(x) + \lim_{x\to 2} f(x) \cdot \lim_{x\to 2} g(x) = 36 \;\Rightarrow\; 3f(2) + f(2)\cdot 6 = 36 \;\Rightarrow\; 9f(2) = 36 \;\Rightarrow\; f(2) = 4.$

14. $\lim_{t\to 1} h(t) = \lim_{t\to 1} \dfrac{2t - 3t^2}{1+t^3} = \dfrac{\lim_{t\to 1}(2t - 3t^2)}{\lim_{t\to 1}(1+t^3)} = \dfrac{2\lim_{t\to 1} t - 3\lim_{t\to 1} t^2}{\lim_{t\to 1} 1 + \lim_{t\to 1} t^3} = \dfrac{2(1) - 3(1)^2}{1+(1)^3} = \dfrac{-1}{2} = h(1).$

By the definition of continuity, h is continuous at $a = 1$.

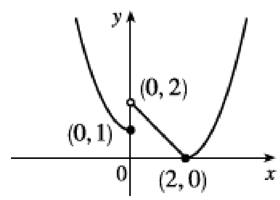
16. For $a < 3$, we have

$$\begin{aligned}
\lim_{x\to a} g(x) &= \lim_{x\to a} 2\sqrt{3-x} \\
&= 2\lim_{x\to a} \sqrt{3-x} && \text{[Limit Law 3]} \\
&= 2\sqrt{\lim_{x\to a}(3-x)} && \text{[11]} \\
&= 2\sqrt{\lim_{x\to a} 3 - \lim_{x\to a} x} && \text{[2]} \\
&= 2\sqrt{3-a} && \text{[7 and 8]} \\
&= g(a)
\end{aligned}$$

So g is continuous at $x = a$ for every a in $(-\infty, 3)$. Also, $\lim_{x\to 3^-} g(x) = 0 = g(3)$, so g is continuous from the left at 3.

Thus, g is continuous on $(-\infty, 3]$.

41. $f(x) = \begin{cases} 1 + x^2 & \text{if } x \le 0 \\ 2 - x & \text{if } 0 < x \le 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$



f is continuous on $(-\infty, 0)$, $(0, 2)$, and $(2, \infty)$ since it is a polynomial on each of these intervals. Now $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(1+x^2) = 1$ and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(2-x) = 2$, so f is discontinuous at 0. Since $f(0) = 1$, f is continuous from the left at 0. Also, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(2-x) = 0$, $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x-2)^2 = 0$, and $f(2) = 0$, so f is continuous at 2. The only number at which f is discontinuous is 0.

44. By Theorem 5, each piece of F is continuous on its domain. We need to check for continuity at $r = R$.

$\lim\limits_{r \to R^-} F(r) = \lim\limits_{r \to R^-} \frac{GMr}{R^3} = \frac{GM}{R^2}$ and $\lim\limits_{r \to R+} F(r) = \lim\limits_{r \to R+} \frac{GM}{r^2} = \frac{GM}{R^2}$, so $\lim\limits_{r \to R} F(r) = \frac{GM}{R^2}$. Since $F(R) = \frac{GM}{R^2}$, F is continuous at R. Therefore, F is a continuous function of r.

65. If there is such a number, it satisfies the equation $x^3 + 1 = x \Leftrightarrow x^3 - x + 1 = 0$. Let the left-hand side of this equation be called $f(x)$. Now $f(-2) = -5 < 0$, and $f(-1) = 1 > 0$. Note also that $f(x)$ is a polynomial, and thus continuous. So by the Intermediate Value Theorem, there is a number c between -2 and -1 such that $f(c) = 0$, so that $c = c^3 + 1$.

3. (a) $\lim\limits_{x \to \infty} f(x) = -2$ (b) $\lim\limits_{x \to -\infty} f(x) = 2$ (c) $\lim\limits_{x \to 1} f(x) = \infty$

(d) $\lim\limits_{x \to 3} f(x) = -\infty$ (e) Vertical: $x = 1$, $x = 3$; horizontal: $y = -2$, $y = 2$

14.

$$\lim_{x \to \infty} \sqrt{\frac{12x^3 - 5x + 2}{1 + 4x^2 + 3x^3}} = \sqrt{\lim_{x \to \infty} \frac{12x^3 - 5x + 2}{1 + 4x^2 + 3x^3}} \qquad \text{[Limit Law 11]}$$

$$= \sqrt{\lim_{x \to \infty} \frac{12 - 5/x^2 + 2/x^3}{1/x^3 + 4/x + 3}} \qquad \text{[divide by } x^3\text{]}$$

$$= \sqrt{\frac{\lim_{x \to \infty} (12 - 5/x^2 + 2/x^3)}{\lim_{x \to \infty} (1/x^3 + 4/x + 3)}} \qquad \text{[Limit Law 5]}$$

$$= \sqrt{\frac{\lim_{x \to \infty} 12 - \lim_{x \to \infty} (5/x^2) + \lim_{x \to \infty} (2/x^3)}{\lim_{x \to \infty} (1/x^3) + \lim_{x \to \infty} (4/x) + \lim_{x \to \infty} 3}} \qquad \text{[Limit Laws 1 and 2]}$$

$$= \sqrt{\frac{12 - 5 \lim_{x \to \infty} (1/x^2) + 2 \lim_{x \to \infty} (1/x^3)}{\lim_{x \to \infty} (1/x^3) + 4 \lim_{x \to \infty} (1/x) + 3}} \qquad \text{[Limit Laws 7 and 3]}$$

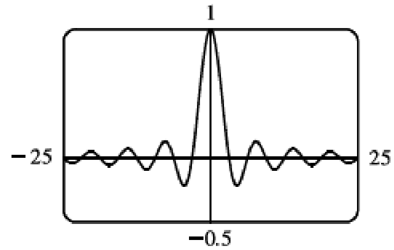
$$= \sqrt{\frac{12 - 5(0) + 2(0)}{0 + 4(0) + 3}} \qquad \text{[Theorem 5 of Section 2.5]}$$

$$= \sqrt{\frac{12}{3}} = \sqrt{4} = 2$$

37. Since $-1 \le \cos x \le 1$ and $e^{-2x} > 0$, we have $-e^{-2x} \le e^{-2x} \cos x \le e^{-2x}$. We know that $\lim\limits_{x \to \infty} (-e^{-2x}) = 0$ and $\lim\limits_{x \to \infty} (e^{-2x}) = 0$, so by the Squeeze Theorem, $\lim\limits_{x \to \infty} (e^{-2x} \cos x) = 0$.

57. (a) Since $-1 \le \sin x \le 1$ for all x, $-\dfrac{1}{x} \le \dfrac{\sin x}{x} \le \dfrac{1}{x}$ for $x > 0$. As $x \to \infty$, $-1/x \to 0$ and $1/x \to 0$, so by the Squeeze Theorem, $(\sin x)/x \to 0$. Thus, $\lim\limits_{x\to\infty} \dfrac{\sin x}{x} = 0$.

(b) From part (a), the horizontal asymptote is $y = 0$. The function $y = (\sin x)/x$ crosses the horizontal asymptote whenever $\sin x = 0$; that is, at $x = \pi n$ for every integer n. Thus, the graph crosses the asymptote *an infinite number of times.*



62. (a) After t minutes, $25t$ liters of brine with 30 g of salt per liter has been pumped into the tank, so it contains $(5000 + 25t)$ liters of water and $25t \cdot 30 = 750t$ grams of salt. Therefore, the salt concentration at time t will be

$$C(t) = \frac{750t}{5000 + 25t} = \frac{30t}{200 + t}\ \frac{\text{g}}{\text{L}}.$$

(b) $\lim\limits_{t\to\infty} C(t) = \lim\limits_{t\to\infty} \dfrac{30t}{200+t} = \lim\limits_{t\to\infty} \dfrac{30t/t}{200/t + t/t} = \dfrac{30}{0+1} = 30$. So the salt concentration approaches that of the brine being pumped into the tank.

8. Using (1) with $f(x) = \dfrac{2x+1}{x+2}$ and $P(1,1)$,

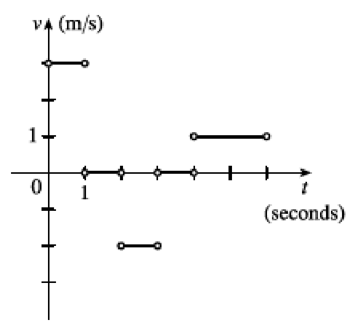
$$m = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} = \lim_{x\to 1} \frac{\dfrac{2x+1}{x+2} - 1}{x - 1} = \lim_{x\to 1} \frac{\dfrac{2x+1-(x+2)}{x+2}}{x-1} = \lim_{x\to 1} \frac{x-1}{(x-1)(x+2)}$$

$$= \lim_{x\to 1} \frac{1}{x+2} = \frac{1}{1+2} = \frac{1}{3}$$

Tangent line: $y - 1 = \frac{1}{3}(x - 1) \quad \Leftrightarrow \quad y - 1 = \frac{1}{3}x - \frac{1}{3} \quad \Leftrightarrow \quad y = \frac{1}{3}x + \frac{2}{3}$

11. (a) The particle is moving to the right when s is increasing; that is, on the intervals $(0, 1)$ and $(4, 6)$. The particle is moving to the left when s is decreasing; that is, on the interval $(2, 3)$. The particle is standing still when s is constant; that is, on the intervals $(1, 2)$ and $(3, 4)$.

(b) The velocity of the particle is equal to the slope of the tangent line of the graph. Note that there is no slope at the corner points on the graph. On the interval $(0, 1)$, the slope is $\dfrac{3-0}{1-0} = 3$. On the interval $(2, 3)$, the slope is $\dfrac{1-3}{3-2} = -2$. On the interval $(4, 6)$, the slope is $\dfrac{3-1}{6-4} = 1$.



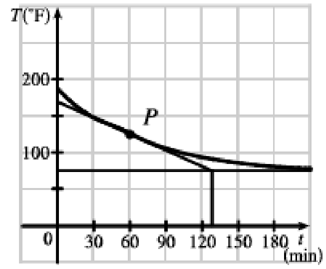
39. $v(5) = f'(5) = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h} = \lim_{h\to 0} \frac{[100 + 50(5+h) - 4.9(5+h)^2] - [100 + 50(5) - 4.9(5)^2]}{h}$

$= \lim_{h\to 0} \frac{(100 + 250 + 50h - 4.9h^2 - 49h - 122.5) - (100 + 250 - 122.5)}{h} = \lim_{h\to 0} \frac{-4.9h^2 + h}{h}$

$= \lim_{h\to 0} \frac{h(-4.9h + 1)}{h} = \lim_{h\to 0}(-4.9h + 1) = 1 \text{ m/s}$

The speed when $t = 5$ is $|1| = 1$ m/s.

42. The slope of the tangent (that is, the rate of change of temperature with respect to time) at $t = 1$ h seems to be about $\frac{75 - 168}{132 - 0} \approx -0.7\,^\circ\text{F/min}$.



51. (a) $S'(T)$ is the rate at which the oxygen solubility changes with respect to the water temperature. Its units are $(\text{mg/L})/^\circ\text{C}$.

(b) For $T = 16^\circ\text{C}$, it appears that the tangent line to the curve goes through the points $(0, 14)$ and $(32, 6)$. So $S'(16) \approx \frac{6 - 14}{32 - 0} = -\frac{8}{32} = -0.25\ (\text{mg/L})/^\circ\text{C}$. This means that as the temperature increases past 16°C, the oxygen solubility is decreasing at a rate of $0.25\ (\text{mg/L})/^\circ\text{C}$.

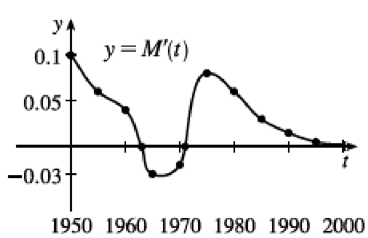
3. (a)$'$ = II, since from left to right, the slopes of the tangents to graph (a) start out negative, become 0, then positive, then 0, then negative again. The actual function values in graph II follow the same pattern.

(b)$'$ = IV, since from left to right, the slopes of the tangents to graph (b) start out at a fixed positive quantity, then suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in the slopes of the tangents.

(c)$'$ = I, since the slopes of the tangents to graph (c) are negative for $x < 0$ and positive for $x > 0$, as are the function values of graph I.

(d)$'$ = III, since from left to right, the slopes of the tangents to graph (d) are positive, then 0, then negative, then 0, then positive, then 0, then negative again, and the function values in graph III follow the same pattern.

15. It appears that there are horizontal tangents on the graph of M for $t = 1963$ and $t = 1971$. Thus, there are zeros for those values of t on the graph of M'. The derivative is negative for the years 1963 to 1971.



25. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^2 - 2(x+h)^3] - (x^2 - 2x^3)}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 2x^3 - 6x^2h - 6xh^2 - 2h^3 - x^2 + 2x^3}{h}$

$= \lim_{h\to 0} \frac{2xh + h^2 - 6x^2h - 6xh^2 - 2h^3}{h} = \lim_{h\to 0} \frac{h(2x + h - 6x^2 - 6xh - 2h^2)}{h}$

$= \lim_{h\to 0} (2x + h - 6x^2 - 6xh - 2h^2) = 2x - 6x^2$

Domain of f = domain of $f' = \mathbb{R}$.

40. f is not differentiable at $x = -1$, because there is a discontinuity there, and at $x = 2$, because the graph has a corner there.