# Calculus 1: Practice Midterm 2

April 2, 2015

Name: _____

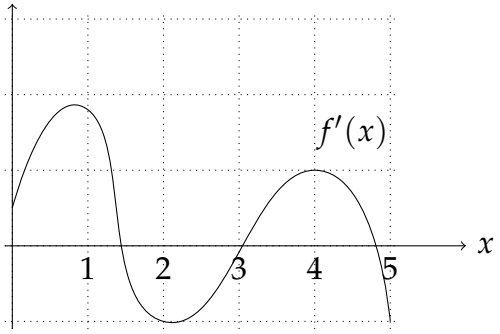
- Write your solutions in the space provided. Continue on the back for more space.
- Show your work. Just writing the final answer will receive little credit.
- Partial credit will be given for incomplete work.
- The exam contains 6 problems.
- The last three pages are left blank for scratch work. You may detach them.
- **Good luck!**

1. Compute the following.

   (a) $f'(x)$ where $f(x) = \sin(2x)\ln(x)$.

   (b) $f''(1)$ where $f(x) = e^{3x} + \dfrac{1}{x}$.

2. Below is the graph of the derivative $f'(x)$ of a function $f(x)$ defined on $(0, 5)$.




(a) What are the critical points of $f(x)$? Which are local minima and which are local maxima?

(b) Find the inflection points of the graph of $f(x)$.

(c) Find an $x$ (any $x$) at which the graph of $f(x)$ is increasing and concave down.

3. Calculate

$$\lim_{x \to +\infty} \left(1 - \frac{1}{x}\right)^x.$$

4. (10 points) The *kinetic energy* of an object is given by the formula

$$K = \frac{1}{2}mv^2,$$

where $m$ is its mass and is $v$ its velocity. The standard unit for $K$ is joules, for $m$ is kilograms, and for $v$ is meters per second.

(a) Suppose a rocket of mass $3 \times 10^6$ kilograms is moving at the speed of $5 \times 10^3$ meters per second and is generating kinetic energy at the rate of $60 \times 10^{11}$ joules per second. At that moment, what the rate of change of its velocity?

(b) Use linear approximation to estimate the velocity after 2 seconds.

5. We want to calculate $\sqrt[3]{4}$ using Newton's method. Write a function whose root is $\sqrt[3]{4}$, and execute the first two steps of Newton's method starting with the initial guess $x_0 = 1$.

6. Suppose Coca Cola were to design its cylindrical Coke can so that it held $100\pi$ ml soda and used the minimum amount of metal. What would be the the radius of the optimum can?