Calculus I: Practice Midterm I

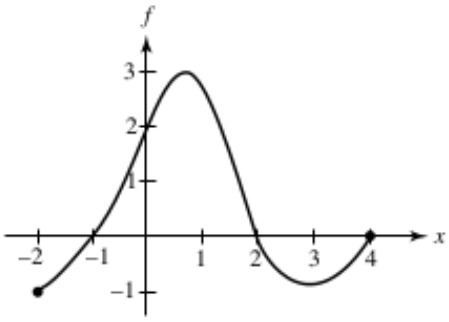
February 13, 2015

Name: _____

- Write your solutions in the space provided. Continue on the back for more space.
- Show your work unless asked otherwise.
- Partial credit will be given for incomplete work.
- The exam contains 6 problems.
- **Good luck!**

Question	Points	Score
1	7	
2	8	
3	9	
4	10	
5	8	
6	8	
Total:	50	

1. Below is the graph of a function f.



(a) (3 points) Use the graph to (approximately) compute the following:
(a) $f(-1)$ and $f(1)$.

(b) All x such that $f(x) = 0$.

(c) The range of f.

(d) (4 points) Let $g(x) = x^2 + 1$. What is $f(g(1))$? What is $g(f(1))$?

2. A 4 foot ladder is leaning against the wall. Denote by x the height of the top end of the ladder (as measured from the floor).

   (a) (3 points) Express the distance of the bottom end of the ladder from the wall as a function of x.

   (b) (3 points) Find the domain and range of the function you found in the previous part.

   (c) (2 points) Draw a rough sketch of the graph of the function.

3. Let

$$f(x) = \frac{e^x}{1+e^x}.$$

It turns out that f has an inverse function.

(a) (3 points) Find $f^{-1}(1/2)$.

(b) (3 points) Find a formula for $f^{-1}(x)$.

(c) (3 points) Write $f(x)$ as the composition of two functions.

4. Calculate each of the following limits, if it exists. Justify your answer.

(a) (3 points) $\lim_{x \to 0} |x| \sin(1/x)$.

(b) (4 points) $\lim_{x \to 1} \dfrac{x^2 - 1}{|x - 1|}$

(c) (3 points) $\lim_{x \to +\infty} \arctan(e^x + 2)$

5. (8 points) Let

$$h(x) = \frac{2x^2 - 3x + 1}{x^2 - 1}$$

Find the horizontal and vertical asymptotes of $h(x)$.

6. Let

$$f(x) = \frac{3x}{1+x}.$$

(a) (6 points) Find $f'(2)$ using the definition of the derivative.

(b) (2 points) Is f increasing or decreasing near $x = 2$?