Calculus 1: Practice Final

May 6, 2015

Name: ____

- Write your solutions in the space provided. Continue on the back for more space.
- Show your work unless asked otherwise.
- Partial credit will be given for incomplete work.
- The exam contains 10 problems.
- **Good luck!**

1. Let

$$f(x) = xe^{-x}.$$

(a) Find $f'(x)$.

(b) Find $f''(x)$.

(c) Is $f(x)$ concave up or concave down at $x = 1$?

2. Let

$$f(x) = x\sqrt{4 - x^2}.$$

(a) Find the domain of f.

(b) Find the global minima and maxima of f.

3. Let

$$f(x) = \frac{x^2+1}{x^2-1}.$$

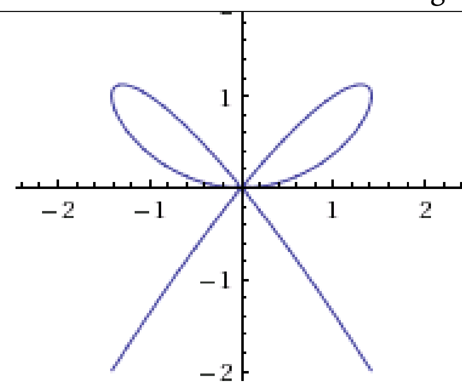
Find the horizontal and vertical asymptotes of the graph of $f(x)$.

4. Evaluate the following

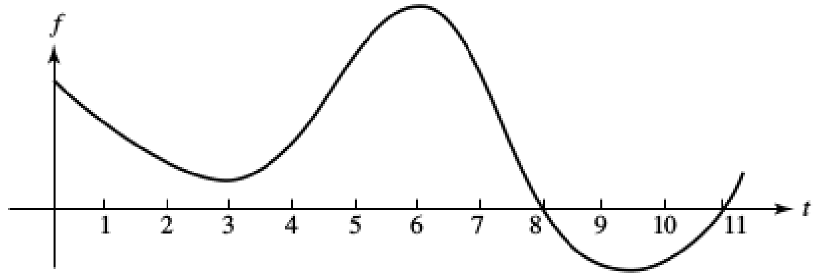
   (a) $f'(x)$ where $f(x) = x^{\sin x}$.

   (b) $\lim_{x \to 0} \dfrac{\sin(3x)\cos(4x)}{\sin(5x)}$

5. The "bow curve" shown here is defined by the equation $x^4 = 3x^2y - 2y^3$. Find the equation of the tangent line to the curve at the point $(1, 1)$.



6. The following is the graph of a function $f(x)$.



(a) Write (but do not evaluate) the Riemann sum for the integral $\int_2^6 f(x)dx$ using 4 parts and left end-points. Draw on the graph the area that the sum represents.

(b) Consider the new function $F(t)$ defined by the formula

$$F(t) = \int_2^{t^2} f(x)\,dx.$$

Determine the sign (positive/negative) of the following quantities:

1. $F(1)$

2. $F'(2)$

3. $F''(3)$

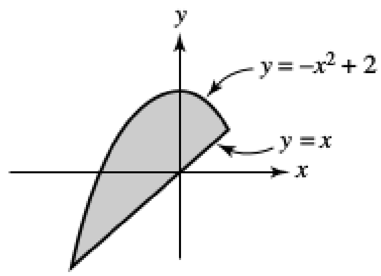
7. Evaluate the integrals

(a) $\displaystyle\int \frac{\ln(x)^2+1}{x}\,dx$

(b) $\displaystyle\int_0^1 \frac{x}{1+x^2}\,dx$

8. Two cars start side by side on parallel roads that are 0.5 miles apart. The first car travels at 30 mph and the second at 40 mph. After one hour, what is the rate of change of the distance between the two cars?

9. Find the shaded (unsigned) area.



10. You are designing a box for blueberries which has a volume of 250 cubic centimeters and a square base. It is made of cardboard except that it has a see-through plastic top. Suppose the plastic costs three times as much as the cardboard. What are the dimensions of the box that minimize the total cost of the materials?

FORMULA SHEET

1. Derivatives

(1) $\dfrac{d\tan x}{dx} = \sec^2 x.$

(2) $\dfrac{d\cot x}{dx} = -\operatorname{cosec}^2 x.$

(3) $\dfrac{d\sec x}{dx} = \sec x \tan x.$

(4) $\dfrac{d\operatorname{cosec} x}{dx} = -\operatorname{cosec} x \cot x.$

(5) $\dfrac{d\arcsin x}{dx} = \dfrac{1}{\sqrt{1-x^2}}.$

(6) $\dfrac{d\arccos x}{dx} = \dfrac{-1}{\sqrt{1-x^2}}.$

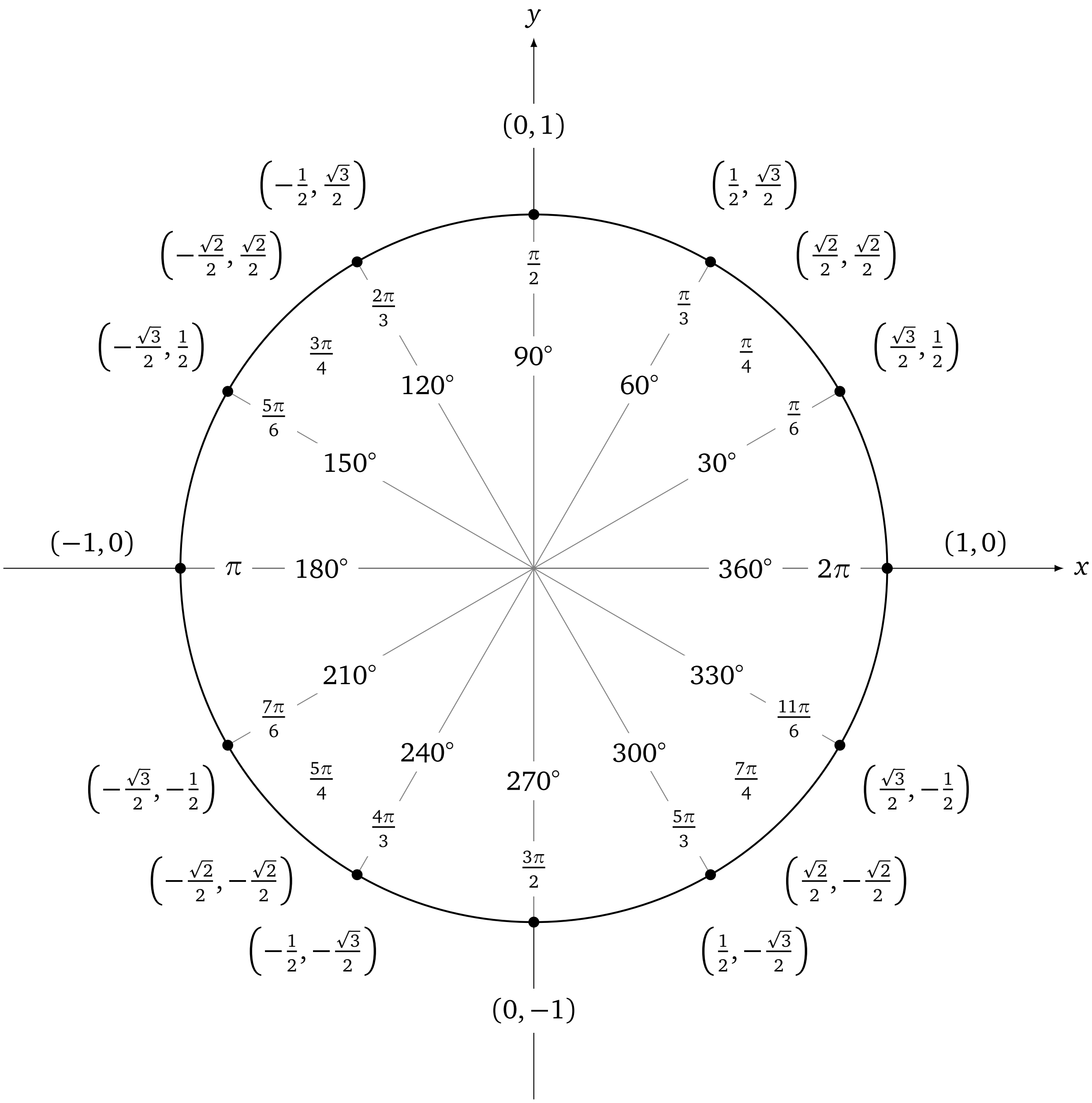
(7) $\dfrac{d\arctan x}{dx} = \dfrac{1}{1+x^2}.$

2. Surface Areas and volumes

(1) Sphere of radius r:
- Volume $= \dfrac{4}{3}\pi r^3$,
- Surface area $= 4\pi r^2$.

(2) Cylinder of radius r and height h:
- Volume $= \pi r^2 h$,
- Curved surface area $= 2\pi r h$,
- Total surface area $= 2\pi r h + 2\pi r^2$.

(3) Cone of radius r and height h:
- Volume $= \dfrac{1}{3}\pi r^2 h$,
- Curved surface area $= \pi r\sqrt{r^2+h^2}$,
- Total surface area $= \pi r\sqrt{r^2+h^2} + \pi r^2$.



y
x
$(0,1)$
$\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$(1,0)$
$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$(0,-1)$
$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$(-1,0)$
$\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{2}$ 90°
$\frac{\pi}{3}$ 60°
$\frac{\pi}{4}$
$\frac{\pi}{6}$ 30°
360° 2π
330° $\frac{11\pi}{6}$
$\frac{7\pi}{4}$
300° $\frac{5\pi}{3}$
270° $\frac{3\pi}{2}$
240° $\frac{4\pi}{3}$
$\frac{5\pi}{4}$
210° $\frac{7\pi}{6}$
π 180°
150° $\frac{5\pi}{6}$
$\frac{3\pi}{4}$
120° $\frac{2\pi}{3}$