Calculus 1: Midterm 2

April 9, 2015

Name: Solutions. (Partial).

Section (Circle one): 8:40–9:55 10:10–11:25

- Write your solutions in the space provided. Continue on the back for more space.
- Show your work unless asked otherwise.
- The exam contains 5 problems.
- The last few pages are for scratch work. You may detach them.
- The very last page is the formula sheet. You may detach it.
- **Good luck!**

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 points) Differentiate the following functions:

(a) $f(x) = \sin(x)e^x$.

✓

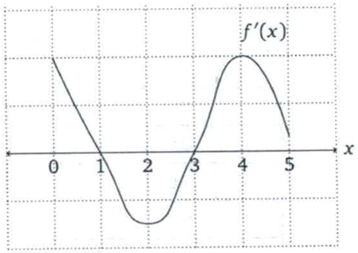
(b) $f(x) = \ln(x^2+1) + x^2 + 1$.

✓

(c) $f(x) = \dfrac{x+1}{x-1}$.

✓

2. The function $f(x)$ is defined on the open interval $(0,5)$. The graph of the derivative of $f(x)$ is given below.



You need not justify your answers in this question.

(a) (2 points) What are the intervals on which $f(x)$ is increasing?

$[0,1]$ $[3,5]$

(b) (2 points) What are the intervals on which $f(x)$ is concave up?

$[2,4]$

(c) (2 points) What are the critical points of $f(x)$?

$x=1$, $x=3$

(d) (2 points) Which are the local minima and which are the local maxima?

$x=1$ is local max , $x=3$ is local min

(e) (2 points) Where is $f''(x)=0$?

$x=2,4$.

3. (10 points) Find

$$\lim_{x\to 0}(1+3x)^{1/x} \quad \text{and} \quad \lim_{x\to+\infty}(1+3x)^{1/x}.$$

Take ln & use L'Hôpital.

$$L = \lim_{x\to 0}(1+3x)^{1/x}$$

$$\ln L = \lim_{x\to 0}\frac{\ln(1+3x)}{x} \quad \text{"}\frac{0}{0}\text{"}$$

$$\ln L = \lim_{x\to 0}\frac{\frac{1}{1+3x}\cdot 3}{1}$$

$$= 3$$

$$L = e^3.$$

Similarly for $x\to+\infty$ we get limit $=1$.

4. (10 points) An oil carrier ship has sprung a leak leading to an oil spill in the Atlantic ocean. The leaked oil forms a thin cylindrical film of height 0.001 meters. At noon, the oil film had a radius of 50 meters and the oil was leaking out at the rate of 3 cubic meters per hour.

(a) At noon, what is the rate of increase of the radius of the spill?

h const

$h = 0.001.$

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} \times h.$$

$$3 = 2\pi \times 50 \times \frac{dr}{dt} \times 0.001$$

$$\frac{dr}{dt} = \frac{30}{\pi} \; m/hr.$$

(b) Use linear approximation to get a rought estimate of the radius at 12:10 pm. You may take $\pi \approx 3$.

radius at 12:10

= radius at 12

$+ \frac{1}{6} \cdot \frac{dr}{dt}$ ← in met./hours

↑

$\frac{1}{6}$ hours

5. (10 points) The sum of two non-negative numbers is 10. What are the maximum and minimum possible values of the sum of their cubes?

$x + y = 10$ $x \geq 0$, $y \geq 0$

min/max $x^3 + y^3$

$f(x) = x^3 + (10-x)^3$ $0 \leq x \leq 10$.

Set $f'(x) = 0$ get $x = 5$

compare $f(0), f(5), f(10)$.

end pts — $f(0)$, $f(10)$

crit pt. — $f(5)$

Scratch work

Scratch work

FORMULA SHEET

1. Derivatives

(1) $\dfrac{d\tan x}{dx} = \sec^2 x.$

(2) $\dfrac{d\cot x}{dx} = -\operatorname{cosec}^2 x.$

(3) $\dfrac{d\sec x}{dx} = \sec x\tan x.$

(4) $\dfrac{d\operatorname{cosec} x}{dx} = -\operatorname{cosec} x\cot x.$

(5) $\dfrac{d\arcsin x}{dx} = \dfrac{1}{\sqrt{1-x^2}}.$

(6) $\dfrac{d\arccos x}{dx} = \dfrac{-1}{\sqrt{1-x^2}}.$

(7) $\dfrac{d\arctan x}{dx} = \dfrac{1}{1+x^2}.$

2. Surface Areas and volumes

(1) Sphere of radius r:

- Volume $= \dfrac{4}{3}\pi r^3$,
- Surface area $= 4\pi r^2$.

(2) Cylinder of radius r and height h:

- Volume $= \pi r^2 h$,
- Curved surface area $= 2\pi r h$,
- Total surface area $= 2\pi r h + 2\pi r^2$.

(3) Cone of radius r and height h:

- Volume $= \dfrac{1}{3}\pi r^2 h$,
- Curved surface area $= \pi r\sqrt{r^2+h^2}$,
- Total surface area $= \pi r\sqrt{r^2+h^2} + \pi r^2$.