Calculus 1: Midterm 2

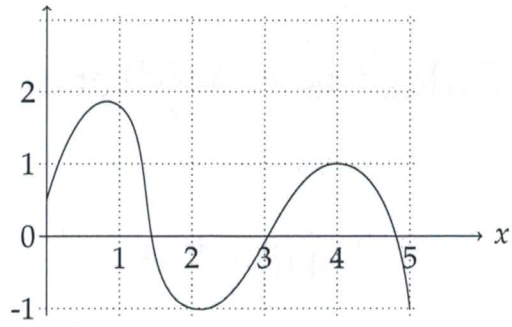
April 3, 2014

Name: Solutions

- Write your solutions in the space provided. Continue on the back for more space.
- Show your work. Just writing the final answer will receive little credit.
- Partial credit will be given for incomplete work.
- The exam contains 5 problems.
- The last three pages are left blank for scratch work. You may detach them.
- **Good luck!**

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. Below is the graph of a function $f(x)$ defined on $[0,5]$.



(a) (3 points) Let $a(x) = xf(x)$. Find $a'(4)$.

$a'(x) = f(x) + x f'(x)$

$a'(4) = f(4) + 4 f'(4)$

$= 2 + 4 \cdot 0 = 2$

(b) (4 points) Let $b(x) = f(x)^2$. For how many x in $[0,5]$ is $b'(x) = 0$?

$b'(x) = 2 f(x) f'(x) = 0$

so $f(x) = 0$ or $f'(x) = 0$

so $x \approx 1.5, 3, 4.8$ or $x \approx 0.8, 2, 4$

(c) (3 points) Which of these numbers are positive: $f(1), f'(1), f''(1)$?

$f(1) > 0$

$f'(1) < 0$ (slope is negative)

$f''(1) < 0$ (concave down)

2. Compute the following.

(a) (5 points) $f'(x)$ where $f(x) = \sin(2x)\ln(x)$.

$$f'(x) = 2\cos(2x)\ln(x) + \frac{\sin(2x)}{x}$$

(b) (5 points) $f''(0)$ where $f(x) = e^{x^2}$.

$$f'(x) = 2x\, e^{x^2}$$

$$f''(x) = 2e^{x^2} + 2x \cdot 2x\, e^{x^2}$$

$$= 2e^{x^2} + 4x^2 e^{x^2}$$

$$f''(0) = 2$$

3. Consider the function defined by

$$f(x) = x^{1/x}.$$

(a) (5 points) Find $\lim_{x\to+\infty} f(x)$. $= L$

$\ln L = \lim_{x\to\infty} \frac{\ln x}{x}$ has form "$\frac{\infty}{\infty}$"

$= \lim_{x\to\infty} \frac{1/x}{1}$ by L'Hôpital.

$\ln L = 0$

so $L = 1 = e^0$

(b) (5 points) Compute $f'(x)$. Use it to find whether $3^{1/3}$ is bigger or smaller than $4^{1/4}$. Justify your answer. You may like to know that $e \approx 2.719$.

[illegible]

When you compute $f'(x)$ you get

$$f'(x) = \frac{x^x}{x^2}(1 - \ln x)$$

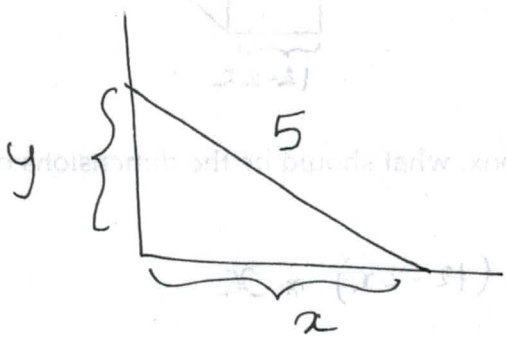
$f'(x)$: 0 e $+\infty$.

f' : $+$ 0 $-$

f ↗ ↘

$\Rightarrow$ f is decreasing on $(e, +\infty)$ so $f(3) > f(4)$ so $3^{1/3} > 4^{1/4}$.

4. (10 points) A five foot ladder is leaning against a wall and starts to slide. At a particular instant, the speed of the top end of the ladder is 2 feet per second and the top end is at the height of 4 feet from the ground. At that instant, what is the speed of the bottom end of the ladder? (As the ladder slides, its top end is always in contact with the wall and its bottom end is always on the ground.)



$y = 4$

$\frac{dy}{dt} = -2$

$x^2 + y^2 = 25$

$x^2 = 25 - y^2 = 9$

$x = 3.$

Diff. $x^2 + y^2 = 25$ w.r.t time t

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$6 \frac{dx}{dt} - 4 \times 2 \times 2 = 0$$

$$\frac{dx}{dt} = \frac{16}{6} = \frac{8}{3} \text{ feet/s}$$

5. (10 points) A machinist is making an open topped box from a flat sheet of metal, which is 12 inches by 12 inches in size. She cuts four square corners and folds the resulting shape to form a box as shown.

To maximize the volume of the resulting box, what should be the dimensions of the cut squares?

Volume = $(12-2x) \times (12-2x) \times x$

$x \in [0, 6]$

max. $(12-2x)^2 x$

diff: $2(12-2x) \times (-2)\, x + (12-2x)^2 \cdot 1 = 0$

$= (12-2x)(-4x + 12 - 2x) = 0$

$(12-2x)(12-6x) = 0$

$12 = 2x$ or $12 = 6x$

Crit pts are: $x = 6$, $x = 2$

End pts: $x = 6, 0.$

Test 0; 2; 6.

$V = 0$ for $x = 0$

$V = 0$ for $x = 6$

$V = 128$ for $x = 2$

max.