1. $y = (x^2 + x^3)^4 \Rightarrow y' = 4(x^2 + x^3)^3(2x + 3x^2) = 4(x^2)^3(1 + x)^3 x(2 + 3x) = 4x^7(x + 1)^3(3x + 2)$

8. $\frac{d}{dx}(xe^y) = \frac{d}{dx}(y \sin x) \Rightarrow xe^y y' + e^y \cdot 1 = y \cos x + \sin x \cdot y' \Rightarrow xe^y y' - \sin x \cdot y' = y \cos x - e^y \Rightarrow$

$(xe^y - \sin x)y' = y \cos x - e^y \Rightarrow y' = \dfrac{y \cos x - e^y}{xe^y - \sin x}$

21. $y = 3^{x \ln x} \Rightarrow y' = 3^{x \ln x}(\ln 3) \frac{d}{dx}(x \ln x) = 3^{x \ln x}(\ln 3)\left(x \cdot \frac{1}{x} + \ln x \cdot 1\right) = 3^{x \ln x}(\ln 3)(1 + \ln x)$

51. $f(t) = \sqrt{4t + 1} \Rightarrow f'(t) = \frac{1}{2}(4t + 1)^{-1/2} \cdot 4 = 2(4t + 1)^{-1/2} \Rightarrow$

$f''(t) = 2(-\frac{1}{2})(4t + 1)^{-3/2} \cdot 4 = -4/(4t + 1)^{3/2}$, so $f''(2) = -4/9^{3/2} = -\frac{4}{27}$.

53. $x^6 + y^6 = 1 \Rightarrow 6x^5 + 6y^5 y' = 0 \Rightarrow y' = -x^5/y^5 \Rightarrow$

$y'' = -\dfrac{y^5(5x^4) - x^5(5y^4 y')}{(y^5)^2} = -\dfrac{5x^4 y^4 \left[y - x(-x^5/y^5)\right]}{y^{10}} = -\dfrac{5x^4 \left[(y^6 + x^6)/y^5\right]}{y^6} = -\dfrac{5x^4}{y^{11}}$

70. (a) $P(x) = f(x)\,g(x) \Rightarrow P'(x) = f(x)\,g'(x) + g(x)\,f'(x) \Rightarrow$

$P'(2) = f(2)\,g'(2) + g(2)\,f'(2) = (1)\left(\frac{6-0}{3-0}\right) + (4)\left(\frac{0-3}{3-0}\right) = (1)(2) + (4)(-1) = 2 - 4 = -2$

(b) $Q(x) = \dfrac{f(x)}{g(x)} \Rightarrow Q'(x) = \dfrac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2} \Rightarrow$

$Q'(2) = \dfrac{g(2)\,f'(2) - f(2)\,g'(2)}{[g(2)]^2} = \dfrac{(4)(-1) - (1)(2)}{4^2} = \dfrac{-6}{16} = -\dfrac{3}{8}$

(c) $C(x) = f(g(x)) \Rightarrow C'(x) = f'(g(x))g'(x) \Rightarrow$

$C'(2) = f'(g(2))g'(2) = f'(4)g'(2) = \left(\frac{6-0}{5-3}\right)(2) = (3)(2) = 6$

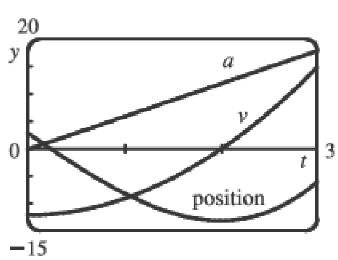
89. (a) $y = t^3 - 12t + 3 \Rightarrow v(t) = y' = 3t^2 - 12 \Rightarrow a(t) = v'(t) = 6t$

(b) $v(t) = 3(t^2 - 4) > 0$ when $t > 2$, so it moves upward when $t > 2$ and downward when $0 \le t < 2$.

(c) Distance upward $= y(3) - y(2) = -6 - (-13) = 7$,

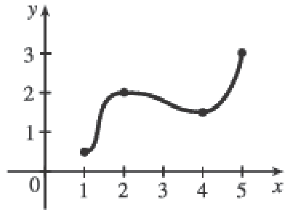
Distance downward $= y(0) - y(2) = 3 - (-13) = 16$. Total distance $= 7 + 16 = 23$.

(d)



(e) The particle is speeding up when v and a have the same sign, that is, when $t > 2$. The particle is slowing down when v and a have opposite signs; that is, when $0 < t < 2$.

8. Absolute minimum at 1, absolute maximum at 5,
local maximum at 2, local minimum at 4



30. $f(x) = x^3 + 6x^2 - 15x \quad \Rightarrow \quad f'(x) = 3x^2 + 12x - 15 = 3(x^2 + 4x - 5) = 3(x+5)(x-1)$.
$f'(x) = 0 \quad \Rightarrow \quad x = -5, 1$. These are the only critical numbers.

40. $g(\theta) = 4\theta - \tan\theta \quad \Rightarrow \quad g'(\theta) = 4 - \sec^2\theta$. $\quad g'(\theta) = 0 \quad \Rightarrow \quad \sec^2\theta = 4 \quad \Rightarrow \quad \sec\theta = \pm 2 \quad \Rightarrow \quad \cos\theta = \pm\frac{1}{2} \quad \Rightarrow$
$\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi$, and $\frac{4\pi}{3} + 2n\pi$ are critical numbers.
Note: The values of θ that make $g'(\theta)$ undefined are not in the domain of g.

41. $f(\theta) = 2\cos\theta + \sin^2\theta \quad \Rightarrow \quad f'(\theta) = -2\sin\theta + 2\sin\theta\,\cos\theta$. $\quad f'(\theta) = 0 \quad \Rightarrow \quad 2\sin\theta\,(\cos\theta - 1) = 0 \quad \Rightarrow \quad \sin\theta = 0$ or $\cos\theta = 1 \quad \Rightarrow \quad \theta = n\pi$ [n an integer] or $\theta = 2n\pi$. The solutions $\theta = n\pi$ include the solutions $\theta = 2n\pi$, so the critical numbers are $\theta = n\pi$.

48. $f(x) = 5 + 54x - 2x^3$, $[0, 4]$. $\quad f'(x) = 54 - 6x^2 = 6(9 - x^2) = 6(3+x)(3-x) = 0 \quad \Leftrightarrow \quad x = -3, 3$. $\quad f(0) = 5$, $f(3) = 113$, and $f(4) = 93$. So $f(3) = 113$ is the absolute maximum value and $f(0) = 5$ is the absolute minimum value.

53. $f(x) = x + \dfrac{1}{x}$, $[0.2, 4]$. $\quad f'(x) = 1 - \dfrac{1}{x^2} = \dfrac{x^2 - 1}{x^2} = \dfrac{(x+1)(x-1)}{x^2} = 0 \quad \Leftrightarrow \quad x = \pm 1$, but $x = -1$ is not in the given interval, $[0.2, 4]$. $f'(x)$ does not exist when $x = 0$, but 0 is not in the given interval, so 1 is the only critical nuumber.
$f(0.2) = 5.2$, $f(1) = 2$, and $f(4) = 4.25$. So $f(0.2) = 5.2$ is the absolute maximum value and $f(1) = 2$ is the absolute minimum value.

6. (a) $f'(x) > 0$ and f is increasing on $(0, 1)$ and $(3, 5)$. $f'(x) < 0$ and f is decreasing on $(1, 3)$ and $(5, 6)$.

(b) Since $f'(x) = 0$ at $x = 1$ and $x = 5$ and f' changes from positive to negative at both values, f changes from increasing to decreasing and has local maxima at $x = 1$ and $x = 5$. Since $f'(x) = 0$ at $x = 3$ and f' changes from negative to positive there, f changes from decreasing to increasing and has a local minimum at $x = 3$.

8. (a) f is increasing on the intervals where $f'(x) > 0$, namely, $(2,4)$ and $(6,9)$.

(b) f has a local maximum where it changes from increasing to decreasing, that is, where f' changes from positive to negative (at $x = 4$). Similarly, where f' changes from negative to positive, f has a local minimum (at $x = 2$ and at $x = 6$).

(c) When f' is increasing, its derivative f'' is positive and hence, f is concave upward. This happens on $(1,3)$, $(5,7)$, and $(8,9)$. Similarly, f is concave downward when f' is decreasing—that is, on $(0,1)$, $(3,5)$, and $(7,8)$.

(d) f has inflection points at $x = 1, 3, 5, 7$, and 8, since the direction of concavity changes at each of these values.

10. (a) $f(x) = 4x^3 + 3x^2 - 6x + 1 \quad \Rightarrow \quad f'(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1) = 6(2x-1)(x+1)$. Thus, $f'(x) > 0 \quad \Leftrightarrow \quad x < -1$ or $x > \frac{1}{2}$ and $f'(x) < 0 \quad \Leftrightarrow \quad -1 < x < \frac{1}{2}$. So f is increasing on $(-\infty, -1)$ and $\left(\frac{1}{2}, \infty\right)$ and f is decreasing on $\left(-1, \frac{1}{2}\right)$.

(b) f changes from increasing to decreasing at $x = -1$ and from decreasing to increasing at $x = \frac{1}{2}$. Thus, $f(-1) = 6$ is a local maximum value and $f\left(\frac{1}{2}\right) = -\frac{3}{4}$ is a local minimum value.

(c) $f''(x) = 24x + 6 = 6(4x+1)$. $\quad f''(x) > 0 \quad \Leftrightarrow \quad x > -\frac{1}{4}$ and $f''(x) < 0 \quad \Leftrightarrow \quad x < -\frac{1}{4}$. Thus, f is concave upward on $\left(-\frac{1}{4}, \infty\right)$ and concave downward on $\left(-\infty, -\frac{1}{4}\right)$. There is an inflection point at $\left(-\frac{1}{4}, f\left(-\frac{1}{4}\right)\right) = \left(-\frac{1}{4}, \frac{21}{8}\right)$.

15. (a) $f(x) = e^{2x} + e^{-x} \quad \Rightarrow \quad f'(x) = 2e^{2x} - e^{-x}$. $\quad f'(x) > 0 \quad \Leftrightarrow \quad 2e^{2x} > e^{-x} \quad \Leftrightarrow \quad e^{3x} > \frac{1}{2} \quad \Leftrightarrow \quad 3x > \ln\frac{1}{2} \quad \Leftrightarrow$ $x > \frac{1}{3}(\ln 1 - \ln 2) \quad \Leftrightarrow \quad x > -\frac{1}{3}\ln 2 \;\; [\approx -0.23]$ and $f'(x) < 0$ if $x < -\frac{1}{3}\ln 2$. So f is increasing on $\left(-\frac{1}{3}\ln 2, \infty\right)$ and f is decreasing on $\left(-\infty, -\frac{1}{3}\ln 2\right)$.

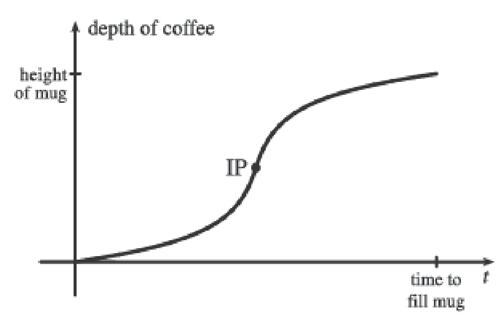
(b) f changes from decreasing to increasing at $x = -\frac{1}{3}\ln 2$. Thus,

$$f\left(-\tfrac{1}{3}\ln 2\right) = f\left(\ln \sqrt[3]{1/2}\right) = e^{2\ln\sqrt[3]{1/2}} + e^{-\ln\sqrt[3]{1/2}} = e^{\ln\sqrt[3]{1/4}} + e^{\ln\sqrt[3]{2}} = \sqrt[3]{1/4} + \sqrt[3]{2} = 2^{-2/3} + 2^{1/3} \;\; [\approx 1.89]$$

is a local minimum value.

(c) $f''(x) = 4e^{2x} + e^{-x} > 0$ [the sum of two positive terms]. Thus, f is concave upward on $(-\infty, \infty)$ and there is no point of inflection.

64. At first the depth increases slowly because the base of the mug is wide. But as the mug narrows, the coffee rises more quickly. Thus, the depth d increases at an increasing rate and its graph is concave upward. The rate of increase of d has a maximum where the mug is narrowest; that is, when the mug is half full. It is there that the inflection point (IP) occurs. Then the rate of increase of d starts to decrease as the mug widens and the graph becomes concave down.



68. $f(x) = axe^{bx^2} \quad \Rightarrow \quad f'(x) = a\left[xe^{bx^2} \cdot 2bx + e^{bx^2} \cdot 1\right] = ae^{bx^2}(2bx^2 + 1)$. For $f(2) = 1$ to be a maximum value, we must have $f'(2) = 0$. $f(2) = 1 \quad \Rightarrow \quad 1 = 2ae^{4b}$ and $f'(2) = 0 \quad \Rightarrow \quad 0 = (8b + 1)ae^{4b}$. So $8b + 1 = 0 \;\; [a \neq 0] \quad \Rightarrow$ $b = -\frac{1}{8}$ and now $1 = 2ae^{-1/2} \quad \Rightarrow \quad a = \sqrt{e}/2$.