3. (a) $\frac{d}{dx}\left(\frac{1}{x}+\frac{1}{y}\right)=\frac{d}{dx}(1) \;\Rightarrow\; -\frac{1}{x^2}-\frac{1}{y^2}\,y'=0 \;\Rightarrow\; -\frac{1}{y^2}\,y'=\frac{1}{x^2} \;\Rightarrow\; y'=-\frac{y^2}{x^2}$

(b) $\frac{1}{x}+\frac{1}{y}=1 \;\Rightarrow\; \frac{1}{y}=1-\frac{1}{x}=\frac{x-1}{x} \;\Rightarrow\; y=\frac{x}{x-1}$, so $y'=\frac{(x-1)(1)-(x)(1)}{(x-1)^2}=\frac{-1}{(x-1)^2}$.

(c) $y'=-\frac{y^2}{x^2}=-\frac{[x/(x-1)]^2}{x^2}=-\frac{x^2}{x^2(x-1)^2}=-\frac{1}{(x-1)^2}$

18. $\frac{d}{dx}(x\sin y+y\sin x)=\frac{d}{dx}(1) \;\Rightarrow\; x\cos y\cdot y'+\sin y\cdot 1+y\cos x+\sin x\cdot y'=0 \;\Rightarrow$

$x\cos y\cdot y'+\sin x\cdot y'=-\sin y-y\cos x \;\Rightarrow\; y'(x\cos y+\sin x)=-\sin y-y\cos x \;\Rightarrow\; y'=\frac{-\sin y-y\cos x}{x\cos y+\sin x}$

32. $y^2(y^2-4)=x^2(x^2-5) \;\Rightarrow\; y^4-4y^2=x^4-5x^2 \;\Rightarrow\; 4y^3\,y'-8y\,y'=4x^3-10x$.

When $x=0$ and $y=-2$, we have $-32y'+16y'=0 \;\Rightarrow\; -16y'=0 \;\Rightarrow\; y'=0$, so an equation of the tangent line is $y+2=0(x-0)$ or $y=-2$.

80. $x^2+4y^2=5 \;\Rightarrow\; 2x+4(2yy')=0 \;\Rightarrow\; y'=-\frac{x}{4y}$. Now let h be the height of the lamp, and let (a,b) be the point of tangency of the line passing through the points $(3,h)$ and $(-5,0)$. This line has slope $(h-0)/[3-(-5)]=\frac{1}{8}h$. But the slope of the tangent line through the point (a,b) can be expressed as $y'=-\frac{a}{4b}$, or as $\frac{b-0}{a-(-5)}=\frac{b}{a+5}$ [since the line passes through $(-5,0)$ and (a,b)], so $-\frac{a}{4b}=\frac{b}{a+5} \;\Leftrightarrow\; 4b^2=-a^2-5a \;\Leftrightarrow\; a^2+4b^2=-5a$. But $a^2+4b^2=5$ [since (a,b) is on the ellipse], so $5=-5a \;\Leftrightarrow\; a=-1$. Then $4b^2=-a^2-5a=-1-5(-1)=4 \;\Rightarrow\; b=1$, since the point is on the top half of the ellipse. So $\frac{h}{8}=\frac{b}{a+5}=\frac{1}{-1+5}=\frac{1}{4} \;\Rightarrow\; h=2$. So the lamp is located 2 units above the x-axis.

6. $y=\frac{1}{\ln x}=(\ln x)^{-1} \;\Rightarrow\; y'=-1(\ln x)^{-2}\cdot\frac{1}{x}=\frac{-1}{x(\ln x)^2}$

24. $y=\frac{\ln x}{x^2} \;\Rightarrow\; y'=\frac{x^2(1/x)-(\ln x)(2x)}{(x^2)^2}=\frac{x(1-2\ln x)}{x^4}=\frac{1-2\ln x}{x^3} \;\Rightarrow$

$y''=\frac{x^3(-2/x)-(1-2\ln x)(3x^2)}{(x^3)^2}=\frac{x^2(-2-3+6\ln x)}{x^6}=\frac{6\ln x-5}{x^4}$

40. $y = \dfrac{e^{-x}\cos^2 x}{x^2+x+1} \quad\Rightarrow\quad \ln y = \ln \dfrac{e^{-x}\cos^2 x}{x^2+x+1} \quad\Rightarrow$

$\ln y = \ln e^{-x} + \ln|\cos x|^2 - \ln(x^2+x+1) = -x + 2\ln|\cos x| - \ln(x^2+x+1) \quad\Rightarrow$

$\dfrac{1}{y}y' = -1 + 2\cdot\dfrac{1}{\cos x}(-\sin x) - \dfrac{1}{x^2+x+1}(2x+1) \quad\Rightarrow\quad y' = y\left(-1 - 2\tan x - \dfrac{2x+1}{x^2+x+1}\right) \quad\Rightarrow$

$y' = -\dfrac{e^{-x}\cos^2 x}{x^2+x+1}\left(1 + 2\tan x + \dfrac{2x+1}{x^2+x+1}\right)$

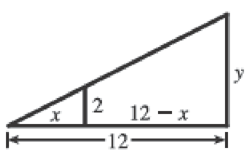
44. $y = x^{\cos x} \quad\Rightarrow\quad \ln y = \ln x^{\cos x} \quad\Rightarrow\quad \ln y = \cos x \ln x \quad\Rightarrow\quad \dfrac{1}{y}y' = \cos x\cdot\dfrac{1}{x} + \ln x\cdot(-\sin x) \quad\Rightarrow$

$y' = y\left(\dfrac{\cos x}{x} - \ln x \sin x\right) \quad\Rightarrow\quad y' = x^{\cos x}\left(\dfrac{\cos x}{x} - \ln x\sin x\right)$

2. (a) $A = \pi r^2 \quad\Rightarrow\quad \dfrac{dA}{dt} = \dfrac{dA}{dr}\dfrac{dr}{dt} = 2\pi r\dfrac{dr}{dt}$ (b) $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt} = 2\pi(30\text{ m})(1\text{ m/s}) = 60\pi\text{ m}^2/\text{s}$

4. $A = \ell w \quad\Rightarrow\quad \dfrac{dA}{dt} = \ell\cdot\dfrac{dw}{dt} + w\cdot\dfrac{d\ell}{dt} = 20(3) + 10(8) = 140\text{ cm}^2/\text{s}.$

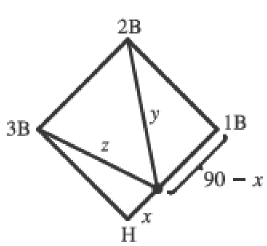
16.



We are given that $\dfrac{dx}{dt} = 1.6$ m/s. By similar triangles, $\dfrac{y}{12} = \dfrac{2}{x} \quad\Rightarrow\quad y = \dfrac{24}{x} \quad\Rightarrow$ $\dfrac{dy}{dt} = -\dfrac{24}{x^2}\dfrac{dx}{dt} = -\dfrac{24}{x^2}(1.6)$. When $x = 8$, $\dfrac{dy}{dt} = -\dfrac{24(1.6)}{64} = -0.6$ m/s, so the shadow is decreasing at a rate of 0.6 m/s.

18. We are given that $\dfrac{dx}{dt} = 24$ ft/s.

(a)



$y^2 = (90-x)^2 + 90^2 \quad\Rightarrow\quad 2y\dfrac{dy}{dt} = 2(90-x)\left(-\dfrac{dx}{dt}\right)$. When $x = 45$,

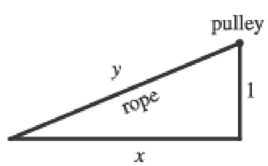
$y = \sqrt{45^2+90^2} = 45\sqrt{5}$, so $\dfrac{dy}{dt} = \dfrac{90-x}{y}\left(-\dfrac{dx}{dt}\right) = \dfrac{45}{45\sqrt{5}}(-24) = -\dfrac{24}{\sqrt{5}}$,

so the distance from second base is decreasing at a rate of $\frac{24}{\sqrt{5}} \approx 10.7$ ft/s.

(b) Due to the symmetric nature of the problem in part (a), we expect to get the same answer—and we do.

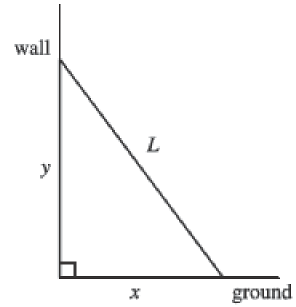
$z^2 = x^2 + 90^2 \quad\Rightarrow\quad 2z\dfrac{dz}{dt} = 2x\dfrac{dx}{dt}$. When $x = 45$, $z = 45\sqrt{5}$, so $\dfrac{dz}{dt} = \dfrac{45}{45\sqrt{5}}(24) = \dfrac{24}{\sqrt{5}} \approx 10.7$ ft/s.

20.



Given $\dfrac{dy}{dt} = -1$ m/s, find $\dfrac{dx}{dt}$ when $x = 8$ m. $y^2 = x^2 + 1 \quad\Rightarrow\quad 2y\dfrac{dy}{dt} = 2x\dfrac{dx}{dt} \quad\Rightarrow$ $\dfrac{dx}{dt} = \dfrac{y}{x}\dfrac{dy}{dt} = -\dfrac{y}{x}$. When $x = 8$, $y = \sqrt{65}$, so $\dfrac{dx}{dt} = -\dfrac{\sqrt{65}}{8}$. Thus, the boat approaches the dock at $\dfrac{\sqrt{65}}{8} \approx 1.01$ m/s.

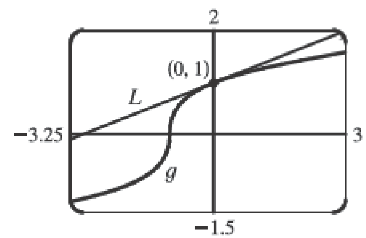
31. From the figure and given information, we have $x^2 + y^2 = L^2$, $\frac{dy}{dt} = -0.15$ m/s, and $\frac{dx}{dt} = 0.2$ m/s when $x = 3$ m. Differentiating implicitly with respect to t, we get $x^2 + y^2 = L^2 \Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow y\frac{dy}{dt} = -x\frac{dx}{dt}$. Substituting the given information gives us $y(-0.15) = -3(0.2) \Rightarrow y = 4$ m. Thus, $3^2 + 4^2 = L^2 \Rightarrow L^2 = 25 \Rightarrow L = 5$ m.



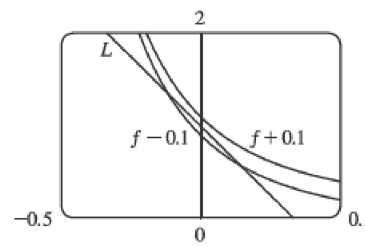
6. $g(x) = \sqrt[3]{1+x} = (1+x)^{1/3} \Rightarrow g'(x) = \frac{1}{3}(1+x)^{-2/3}$, so $g(0) = 1$ and $g'(0) = \frac{1}{3}$. Therefore, $\sqrt[3]{1+x} = g(x) \approx g(0) + g'(0)(x-0) = 1 + \frac{1}{3}x$.

So $\sqrt[3]{0.95} = \sqrt[3]{1+(-0.05)} \approx 1 + \frac{1}{3}(-0.05) = 0.98\overline{3}$,

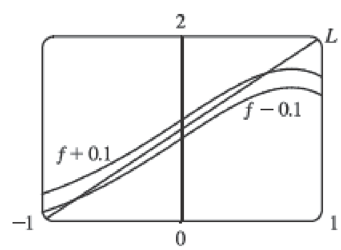
and $\sqrt[3]{1.1} = \sqrt[3]{1+0.1} \approx 1 + \frac{1}{3}(0.1) = 1.0\overline{3}$.



8. $f(x) = (1+x)^{-3} \Rightarrow f'(x) = -3(1+x)^{-4}$, so $f(0) = 1$ and $f'(0) = -3$. Thus, $f(x) \approx f(0) + f'(0)(x-0) = 1 - 3x$. We need $(1+x)^{-3} - 0.1 < 1 - 3x < (1+x)^{-3} + 0.1$, which is true when $-0.116 < x < 0.144$.



10. $f(x) = e^x \cos x \Rightarrow f'(x) = e^x(-\sin x) + (\cos x)e^x = e^x(\cos x - \sin x)$, so $f(0) = 1$ and $f'(0) = 1$. Thus, $f(x) \approx f(0) + f'(0)(x-0) = 1 + x$. We need $e^x \cos x - 0.1 < 1 + x < e^x \cos x + 0.1$, which is true when $-0.762 < x < 0.607$.



25. $y = f(x) = \sqrt[3]{x} \Rightarrow dy = \frac{1}{3}x^{-2/3}\,dx$. When $x = 1000$ and $dx = 1$, $dy = \frac{1}{3}(1000)^{-2/3}(1) = \frac{1}{300}$, so $\sqrt[3]{1001} = f(1001) \approx f(1000) + dy = 10 + \frac{1}{300} = 10.00\overline{3} \approx 10.003$.

28. $y = f(x) = \sqrt{x} \Rightarrow dy = \frac{1}{2\sqrt{x}}\,dx$. When $x = 100$ and $dx = -0.2$, $dy = \frac{1}{2\sqrt{100}}(-0.2) = -0.01$, so $\sqrt{99.8} = f(99.8) \approx f(100) + dy = 10 - 0.01 = 9.99$.