1. Product Rule: $f(x) = (1+2x^2)(x-x^2) \quad \Rightarrow$

$$f'(x) = (1+2x^2)(1-2x) + (x-x^2)(4x) = 1 - 2x + 2x^2 - 4x^3 + 4x^2 - 4x^3 = 1 - 2x + 6x^2 - 8x^3.$$

Multiplying first: $f(x) = (1+2x^2)(x-x^2) = x - x^2 + 2x^3 - 2x^4 \quad \Rightarrow \quad f'(x) = 1 - 2x + 6x^2 - 8x^3$ (equivalent).

4. By the Product Rule, $g(x) = \sqrt{x}\, e^x = x^{1/2} e^x \quad \Rightarrow \quad g'(x) = x^{1/2}(e^x) + e^x\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}e^x(2x+1)$.

6. By the Quotient Rule, $y = \dfrac{e^x}{1-e^x} \quad \Rightarrow \quad y' = \dfrac{(1-e^x)e^x - e^x(-e^x)}{(1-e^x)^2} = \dfrac{e^x - e^{2x} + e^{2x}}{(1-e^x)^2} = \dfrac{e^x}{(1-e^x)^2}$.

27. $f(x) = x^4 e^x \quad \Rightarrow \quad f'(x) = x^4 e^x + e^x \cdot 4x^3 = (x^4 + 4x^3)e^x \;\; [\text{or } x^3 e^x (x+4)] \quad \Rightarrow$

$$f''(x) = (x^4 + 4x^3)e^x + e^x(4x^3 + 12x^2) = (x^4 + 4x^3 + 4x^3 + 12x^2)e^x$$
$$= (x^4 + 8x^3 + 12x^2)e^x \;\; [\text{or } x^2 e^x (x+2)(x+6)]$$

32. $y = \dfrac{e^x}{x} \quad \Rightarrow \quad y' = \dfrac{x \cdot e^x - e^x \cdot 1}{x^2} = \dfrac{e^x(x-1)}{x^2}$.

At $(1, e)$, $y' = 0$, and an equation of the tangent line is $y - e = 0(x-1)$, or $y = e$.

44. We are given that $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$, and $g'(2) = 7$.

(a) $h(x) = 5f(x) - 4g(x) \quad \Rightarrow \quad h'(x) = 5f'(x) - 4g'(x)$, so

$h'(2) = 5f'(2) - 4g'(2) = 5(-2) - 4(7) = -10 - 28 = -38$.

(b) $h(x) = f(x)g(x) \quad \Rightarrow \quad h'(x) = f(x)g'(x) + g(x)f'(x)$, so

$h'(2) = f(2)g'(2) + g(2)f'(2) = (-3)(7) + (4)(-2) = -21 - 8 = -29$.

(c) $h(x) = \dfrac{f(x)}{g(x)} \quad \Rightarrow \quad h'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$, so

$$h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} = \frac{4(-2) - (-3)(7)}{4^2} = \frac{-8+21}{16} = \frac{13}{16}.$$

(d) $h(x) = \dfrac{g(x)}{1+f(x)} \quad \Rightarrow \quad h'(x) = \dfrac{[1+f(x)]\, g'(x) - g(x)f'(x)}{[1+f(x)]^2}$, so

$$h'(2) = \frac{[1+f(2)]\, g'(2) - g(2)\, f'(2)}{[1+f(x)]^2} = \frac{[1+(-3)](7) - 4(-2)}{[1+(-3)]^2} = \frac{-14+8}{(-2)^2} = \frac{-6}{4} = -\frac{3}{2}.$$

50. (a) $P(x) = F(x)\, G(x)$, so $P'(2) = F(2)\, G'(2) + G(2)\, F'(2) = 3 \cdot \frac{2}{4} + 2 \cdot 0 = \frac{3}{2}$.

(b) $Q(x) = F(x)/G(x)$, so $Q'(7) = \dfrac{G(7)\, F'(7) - F(7)\, G'(7)}{[G(7)]^2} = \dfrac{1 \cdot \frac{1}{4} - 5 \cdot \left(-\frac{2}{3}\right)}{1^2} = \dfrac{1}{4} + \dfrac{10}{3} = \dfrac{43}{12}$

2. $f(x) = \sqrt{x}\sin x \quad \Rightarrow \quad f'(x) = \sqrt{x}\cos x + \sin x\left(\frac{1}{2}x^{-1/2}\right) = \sqrt{x}\cos x + \dfrac{\sin x}{2\sqrt{x}}$

4. $y = 2\sec x - \csc x \quad \Rightarrow \quad y' = 2(\sec x\,\tan x) - (-\csc x\,\cot x) = 2\sec x\,\tan x + \csc x\,\cot x$

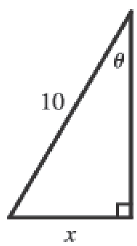
10. $y = \sin\theta\,\cos\theta \quad \Rightarrow \quad y' = \sin\theta(-\sin\theta) + \cos\theta(\cos\theta) = \cos^2\theta - \sin^2\theta \quad$ [or $\cos 2\theta$]

12. $y = \dfrac{\cos x}{1 - \sin x} \quad \Rightarrow$

$$y' = \frac{(1-\sin x)(-\sin x) - \cos x(-\cos x)}{(1-\sin x)^2} = \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$$
$$= \frac{-\sin x + 1}{(1-\sin x)^2} = \frac{1}{1-\sin x}$$

24. $y = x + \tan x \quad \Rightarrow \quad y' = 1 + \sec^2 x$, so $y'(\pi) = 1 + (-1)^2 = 2$. An equation of the tangent line to the curve $y = x + \tan x$ at the point (π, π) is $y - \pi = 2(x - \pi)$ or $y = 2x - \pi$.

37.



From the diagram we can see that $\sin\theta = x/10 \quad \Leftrightarrow \quad x = 10\sin\theta$. We want to find the rate of change of x with respect to θ, that is, $dx/d\theta$. Taking the derivative of $x = 10\sin\theta$, we get $dx/d\theta = 10(\cos\theta)$. So when $\theta = \frac{\pi}{3}$, $\frac{dx}{d\theta} = 10\cos\frac{\pi}{3} = 10\left(\frac{1}{2}\right) = 5$ ft/rad.

40. $\displaystyle\lim_{x\to 0}\frac{\sin 4x}{\sin 6x} = \lim_{x\to 0}\left(\frac{\sin 4x}{x}\cdot\frac{x}{\sin 6x}\right) = \lim_{x\to 0}\frac{4\sin 4x}{4x}\cdot\lim_{x\to 0}\frac{6x}{6\sin 6x} = 4\lim_{x\to 0}\frac{\sin 4x}{4x}\cdot\frac{1}{6}\lim_{x\to 0}\frac{6x}{\sin 6x} = 4(1)\cdot\frac{1}{6}(1) = \frac{2}{3}$

2. Let $u = g(x) = 2x^3 + 5$ and $y = f(u) = u^4$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (4u^3)(6x^2) = 24x^2(2x^3+5)^3$.

24. Using Formula 5 and the Chain Rule, $y = 10^{1-x^2} \quad \Rightarrow \quad y' = 10^{1-x^2}(\ln 10)\cdot\dfrac{d}{dx}\left(1 - x^2\right) = -2x(\ln 10)10^{1-x^2}$.

40. $y = \sin(\sin(\sin x)) \quad \Rightarrow \quad y' = \cos(\sin(\sin x))\,\dfrac{d}{dx}(\sin(\sin x)) = \cos(\sin(\sin x))\,\cos(\sin x)\,\cos x$

51. $y = (1+2x)^{10} \quad \Rightarrow \quad y' = 10(1+2x)^9\cdot 2 = 20(1+2x)^9$.

At $(0, 1)$, $y' = 20(1+0)^9 = 20$, and an equation of the tangent line is $y - 1 = 20(x - 0)$, or $y = 20x + 1$.

63. (a) $h(x) = f(g(x)) \quad \Rightarrow \quad h'(x) = f'(g(x))\cdot g'(x)$, so $h'(1) = f'(g(1))\cdot g'(1) = f'(2)\cdot 6 = 5\cdot 6 = 30$.

(b) $H(x) = g(f(x)) \quad \Rightarrow \quad H'(x) = g'(f(x))\cdot f'(x)$, so $H'(1) = g'(f(1))\cdot f'(1) = g'(3)\cdot 4 = 9\cdot 4 = 36$.

67. The point $(3, 2)$ is on the graph of f, so $f(3) = 2$. The tangent line at $(3, 2)$ has slope $\dfrac{\Delta y}{\Delta x} = \dfrac{-4}{6} = -\dfrac{2}{3}$.

$g(x) = \sqrt{f(x)} \quad \Rightarrow \quad g'(x) = \frac{1}{2}[f(x)]^{-1/2} \cdot f'(x) \quad \Rightarrow$

$g'(3) = \frac{1}{2}[f(3)]^{-1/2} \cdot f'(3) = \frac{1}{2}(2)^{-1/2}(-\frac{2}{3}) = -\dfrac{1}{3\sqrt{2}}$ or $-\frac{1}{6}\sqrt{2}$.