2. (a) Slope $= \frac{2948-2530}{42-36} = \frac{418}{6} \approx 69.67$

(b) Slope $= \frac{2948-2661}{42-38} = \frac{287}{4} = 71.75$

(c) Slope $= \frac{2948-2806}{42-40} = \frac{142}{2} = 71$

(d) Slope $= \frac{3080-2948}{44-42} = \frac{132}{2} = 66$

From the data, we see that the patient's heart rate is decreasing from 71 to 66 heartbeats/minute after 42 minutes. After being stable for a while, the patient's heart rate is dropping.

6. (a) $h(x)$ approaches 4 as x approaches -3 from the left, so $\lim_{x\to -3^-} h(x) = 4$.

(b) $h(x)$ approaches 4 as x approaches -3 from the right, so $\lim_{x\to -3^+} h(x) = 4$.

(c) $\lim_{x\to -3} h(x) = 4$ because the limits in part (a) and part (b) are equal.

(d) $h(-3)$ is not defined, so it doesn't exist.

(e) $h(x)$ approaches 1 as x approaches 0 from the left, so $\lim_{x\to 0^-} h(x) = 1$.

(f) $h(x)$ approaches -1 as x approaches 0 from the right, so $\lim_{x\to 0^+} h(x) = -1$.

(g) $\lim_{x\to 0} h(x)$ does not exist because the limits in part (e) and part (f) are not equal.

(h) $h(0) = 1$ since the point $(0, 1)$ is on the graph of h.

(i) Since $\lim_{x\to 2^-} h(x) = 2$ and $\lim_{x\to 2^+} h(x) = 2$, we have $\lim_{x\to 2} h(x) = 2$.

(j) $h(2)$ is not defined, so it doesn't exist.

(k) $h(x)$ approaches 3 as x approaches 5 from the right, so $\lim_{x\to 5^+} h(x) = 3$.

(l) $h(x)$ does not approach any one number as x approaches 5 from the left, so $\lim_{x\to 5^-} h(x)$ does not exist.

8. (a) $\lim_{x\to 2} R(x) = -\infty$

(b) $\lim_{x\to 5} R(x) = \infty$

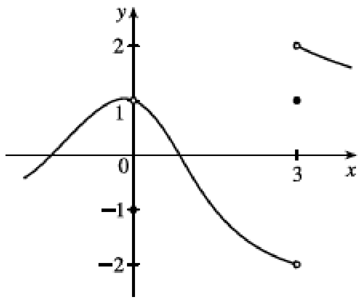
(c) $\lim_{x\to -3^-} R(x) = -\infty$

(d) $\lim_{x\to -3^+} R(x) = \infty$

(e) The equations of the vertical asymptotes are $x = -3$, $x = 2$, and $x = 5$.

16. $\lim_{x\to 0} f(x) = 1$, $\lim_{x\to 3^-} f(x) = -2$, $\lim_{x\to 3^+} f(x) = 2$,

$f(0) = -1$, $f(3) = 1$



2. (a) $\lim_{x\to 2}[f(x)+g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 2 + 0 = 2$

(b) $\lim_{x\to 1} g(x)$ does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

(c) $\lim_{x\to 0}[f(x)g(x)] = \lim_{x\to 0} f(x) \cdot \lim_{x\to 0} g(x) = 0 \cdot 1.3 = 0$

(d) Since $\lim_{x\to -1} g(x) = 0$ and g is in the denominator, but $\lim_{x\to -1} f(x) = -1 \neq 0$, the given limit does not exist.

(e) $\lim_{x\to 2} x^3 f(x) = \left[\lim_{x\to 2} x^3\right]\left[\lim_{x\to 2} f(x)\right] = 2^3 \cdot 2 = 16$

(f) $\lim_{x\to 1}\sqrt{3+f(x)} = \sqrt{3+\lim_{x\to 1} f(x)} = \sqrt{3+1} = 2$

4.
$$\begin{aligned}
\lim_{x\to -1}(x^4-3x)(x^2+5x+3) &= \lim_{x\to -1}(x^4-3x)\lim_{x\to -1}(x^2+5x+3) && \text{[Limit Law 4]}\\
&= \left(\lim_{x\to -1} x^4 - \lim_{x\to -1} 3x\right)\left(\lim_{x\to -1} x^2 + \lim_{x\to -1} 5x + \lim_{x\to -1} 3\right) && \text{[2, 1]}\\
&= \left(\lim_{x\to -1} x^4 - 3\lim_{x\to -1} x\right)\left(\lim_{x\to -1} x^2 + 5\lim_{x\to -1} x + \lim_{x\to -1} 3\right) && \text{[3]}\\
&= (1+3)(1-5+3) && \text{[9, 8, and 7]}\\
&= 4(-1) = -4
\end{aligned}$$

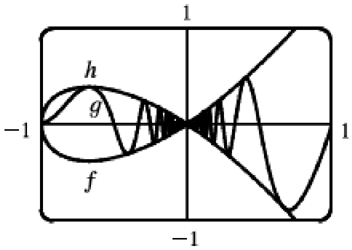
12. $\lim_{x\to 4}\frac{x^2-4x}{x^2-3x-4} = \lim_{x\to 4}\frac{x(x-4)}{(x-4)(x+1)} = \lim_{x\to 4}\frac{x}{x+1} = \frac{4}{4+1} = \frac{4}{5}$

14. $\lim_{x\to -1}\frac{x^2-4x}{x^2-3x-4}$ does not exist since $x^2-3x-4 \to 0$ but $x^2-4x \to 5$ as $x \to -1$.

23. $\lim_{x\to -4}\frac{\frac{1}{4}+\frac{1}{x}}{4+x} = \lim_{x\to -4}\frac{\frac{x+4}{4x}}{4+x} = \lim_{x\to -4}\frac{x+4}{4x(4+x)} = \lim_{x\to -4}\frac{1}{4x} = \frac{1}{4(-4)} = -\frac{1}{16}$

26. $\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right) = \lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t(t+1)}\right) = \lim_{t\to 0}\frac{t+1-1}{t(t+1)} = \lim_{t\to 0}\frac{1}{t+1} = \frac{1}{0+1} = 1$

36. Let $f(x) = -\sqrt{x^3+x^2}$, $g(x) = \sqrt{x^3+x^2}\,\sin(\pi/x)$, and $h(x) = \sqrt{x^3+x^2}$. Then $-1 \le \sin(\pi/x) \le 1 \;\Rightarrow\; -\sqrt{x^3+x^2} \le \sqrt{x^3+x^2}\,\sin(\pi/x) \le \sqrt{x^3+x^2} \;\Rightarrow\; f(x) \le g(x) \le h(x)$. So since $\lim_{x\to 0} f(x) = \lim_{x\to 0} h(x) = 0$, by the Squeeze Theorem we have $\lim_{x\to 0} g(x) = 0$.



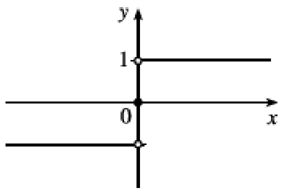
42. $|x+6| = \begin{cases} x+6 & \text{if } x+6 \geq 0 \\ -(x+6) & \text{if } x+6 < 0 \end{cases} = \begin{cases} x+6 & \text{if } x \geq -6 \\ -(x+6) & \text{if } x < -6 \end{cases}$

We'll look at the one-sided limits.

$$\lim_{x\to -6^+} \frac{2x+12}{|x+6|} = \lim_{x\to -6^+} \frac{2(x+6)}{x+6} = 2 \quad \text{and} \quad \lim_{x\to -6^-} \frac{2x+12}{|x+6|} = \lim_{x\to -6^-} \frac{2(x+6)}{-(x+6)} = -2$$

The left and right limits are different, so $\lim_{x\to -6} \frac{2x+12}{|x+6|}$ does not exist.

47. (a)



(b) (i) Since $\operatorname{sgn} x = 1$ for $x > 0$, $\lim_{x\to 0^+} \operatorname{sgn} x = \lim_{x\to 0^+} 1 = 1$.

(ii) Since $\operatorname{sgn} x = -1$ for $x < 0$, $\lim_{x\to 0^-} \operatorname{sgn} x = \lim_{x\to 0^-} -1 = -1$.

(iii) Since $\lim_{x\to 0^-} \operatorname{sgn} x \neq \lim_{x\to 0^+} \operatorname{sgn} x$, $\lim_{x\to 0} \operatorname{sgn} x$ does not exist.

(iv) Since $|\operatorname{sgn} x| = 1$ for $x \neq 0$, $\lim_{x\to 0} |\operatorname{sgn} x| = \lim_{x\to 0} 1 = 1$.

4. (a) $\lim_{x\to\infty} g(x) = 2$ (b) $\lim_{x\to -\infty} g(x) = -1$ (c) $\lim_{x\to 0} g(x) = -\infty$

(d) $\lim_{x\to 2^-} g(x) = -\infty$ (e) $\lim_{x\to 2^+} g(x) = \infty$ (f) Vertical: $x = 0$, $x = 2$;
horizontal: $y = -1$, $y = 2$

16. $$\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1} = \lim_{x\to\infty} \frac{(1-x^2)/x^3}{(x^3-x+1)/x^3} = \lim_{x\to\infty} \frac{1/x^3 - 1/x}{1 - 1/x^2 + 1/x^3}$$
$$= \frac{\lim_{x\to\infty} 1/x^3 - \lim_{x\to\infty} 1/x}{\lim_{x\to\infty} 1 - \lim_{x\to\infty} 1/x^2 + \lim_{x\to\infty} 1/x^3} = \frac{0-0}{1-0+0} = 0$$

22. $$\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+1}} = \lim_{x\to\infty} \frac{x^2/x^2}{\sqrt{x^4+1}/x^2} = \lim_{x\to\infty} \frac{1}{\sqrt{(x^4+1)/x^4}} \qquad [\text{since } x^2 = \sqrt{x^4} \text{ for } x > 0]$$
$$= \lim_{x\to\infty} \frac{1}{\sqrt{1+1/x^4}} = \frac{1}{\sqrt{1+0}} = 1$$

30. $\lim_{x\to\infty} (e^{-x} + 2\cos 3x)$ does not exist. $\lim_{x\to\infty} e^{-x} = 0$, but $\lim_{x\to\infty} (2\cos 3x)$ does not exist because the values of $2\cos 3x$ oscillate between the values of -2 and 2 infinitely often, so the given limit does not exist.

33. Let $t = e^x$. As $x \to \infty$, $t \to \infty$. $\lim_{x\to\infty} \arctan(e^x) = \lim_{t\to\infty} \arctan t = \frac{\pi}{2}$ by (3).

43. $\lim\limits_{x\to\infty} \dfrac{2x^2+x-1}{x^2+x-2} = \lim\limits_{x\to\infty} \dfrac{\dfrac{2x^2+x-1}{x^2}}{\dfrac{x^2+x-2}{x^2}} = \lim\limits_{x\to\infty} \dfrac{2+\dfrac{1}{x}-\dfrac{1}{x^2}}{1+\dfrac{1}{x}-\dfrac{2}{x^2}} = \dfrac{\lim\limits_{x\to\infty}\left(2+\dfrac{1}{x}-\dfrac{1}{x^2}\right)}{\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}-\dfrac{2}{x^2}\right)}$

$= \dfrac{\lim\limits_{x\to\infty} 2 + \lim\limits_{x\to\infty}\dfrac{1}{x} - \lim\limits_{x\to\infty}\dfrac{1}{x^2}}{\lim\limits_{x\to\infty} 1 + \lim\limits_{x\to\infty}\dfrac{1}{x} - 2\lim\limits_{x\to\infty}\dfrac{1}{x^2}} = \dfrac{2+0-0}{1+0-2(0)} = 2,$ so $y = 2$ is a horizontal asymptote.

$y = f(x) = \dfrac{2x^2+x-1}{x^2+x-2} = \dfrac{(2x-1)(x+1)}{(x+2)(x-1)}$, so $\lim\limits_{x\to-2^-} f(x) = \infty$,

$\lim\limits_{x\to-2^+} f(x) = -\infty$, $\lim\limits_{x\to1^-} f(x) = -\infty$, and $\lim\limits_{x\to1^+} f(x) = \infty$. Thus, $x = -2$

and $x = 1$ are vertical asymptotes. The graph confirms our work.