1. $\frac{d}{dx}\left[-\frac{\sqrt{1+x^2}}{x}+C\right]=\frac{d}{dx}\left[-\frac{(1+x^2)^{1/2}}{x}+C\right]=-\frac{x\cdot\frac{1}{2}(1+x^2)^{-1/2}(2x)-(1+x^2)^{1/2}\cdot 1}{(x)^2}+0$

$$=-\frac{(1+x^2)^{-1/2}\left[x^2-(1+x^2)\right]}{x^2}=-\frac{-1}{(1+x^2)^{1/2}x^2}=\frac{1}{x^2\sqrt{1+x^2}}$$

6. $\int\left(\sqrt{x^3}+\sqrt[3]{x^2}\right)dx=\int(x^{3/2}+x^{2/3})\,dx=\frac{x^{5/2}}{5/2}+\frac{x^{5/3}}{5/3}+C=\frac{2}{5}x^{5/2}+\frac{3}{5}x^{5/3}+C$

10. $\int v(v^2+2)^2\,dv=\int v(v^4+4v^2+4)\,dv=\int(v^5+4v^3+4v)\,dv=\frac{v^6}{6}+4\frac{v^4}{4}+4\frac{v^2}{2}+C=\frac{1}{6}v^6+v^4+2v^2+C$

22. $\int_1^2(4x^3-3x^2+2x)\,dx=\left[x^4-x^3+x^2\right]_1^2=(16-8+4)-(1-1+1)=12-1=11$

37. $\int_0^{\pi/4}\frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta=\int_0^{\pi/4}\left(\frac{1}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}\right)d\theta=\int_0^{\pi/4}(\sec^2\theta+1)\,d\theta$

$$=\left[\tan\theta+\theta\right]_0^{\pi/4}=\left(\tan\frac{\pi}{4}+\frac{\pi}{4}\right)-(0+0)=1+\frac{\pi}{4}$$

2. Let $u=2+x^4$. Then $du=4x^3\,dx$ and $x^3\,dx=\frac{1}{4}\,du$,

so $\int x^3(2+x^4)^5\,dx=\int u^5\left(\frac{1}{4}\,du\right)=\frac{1}{4}\frac{u^6}{6}+C=\frac{1}{24}(2+x^4)^6+C$.

5. Let $u=\cos\theta$. Then $du=-\sin\theta\,d\theta$ and $\sin\theta\,d\theta=-du$, so

$\int\cos^3\theta\,\sin\theta\,d\theta=\int u^3\,(-du)=-\frac{u^4}{4}+C=-\frac{1}{4}\cos^4\theta+C$.

7. Let $u=x^2$. Then $du=2x\,dx$ and $x\,dx=\frac{1}{2}\,du$, so $\int x\sin(x^2)\,dx=\int\sin u\left(\frac{1}{2}\,du\right)=-\frac{1}{2}\cos u+C=-\frac{1}{2}\cos(x^2)+C$.

8. Let $u=x^3$. Then $du=3x^2\,dx$ and $x^2\,dx=\frac{1}{3}\,du$, so $\int x^2e^{x^3}\,dx=\int e^u\left(\frac{1}{3}\,du\right)=\frac{1}{3}e^u+C=\frac{1}{3}e^{x^3}+C$.

16. Let $u=e^x$. Then $du=e^x\,dx$, so $\int e^x\cos(e^x)\,dx=\int\cos u\,du=\sin u+C=\sin(e^x)+C$.

21. Let $u=\ln x$. Then $du=\frac{dx}{x}$, so $\int\frac{(\ln x)^2}{x}\,dx=\int u^2\,du=\frac{1}{3}u^3+C=\frac{1}{3}(\ln x)^3+C$.

41. $\int\cot x\,dx=\int\frac{\cos x}{\sin x}\,dx$. Let $u=\sin x$. Then $du=\cos x\,dx$, so $\int\cot x\,dx=\int\frac{1}{u}\,du=\ln|u|+C=\ln|\sin x|+C$.

78. Let $u=x^2$. Then $du=2x\,dx$ and the limits are unchanged ($0^2=0$ and $1^2=1$), so

$I=\int_0^1 x\sqrt{1-x^4}\,dx=\frac{1}{2}\int_0^1\sqrt{1-u^2}\,du$. But this integral can be interpreted as the area of a quarter-circle with radius 1.

So $I=\frac{1}{2}\cdot\frac{1}{4}(\pi\cdot 1^2)=\frac{1}{8}\pi$.

86. Let $u = x^2$. Then $du = 2x\,dx$, so $\int_0^3 x f(x^2)\,dx = \int_0^9 f(u)\left(\frac{1}{2}\,du\right) = \frac{1}{2}\int_0^9 f(u)\,du = \frac{1}{2}(4) = 2$.

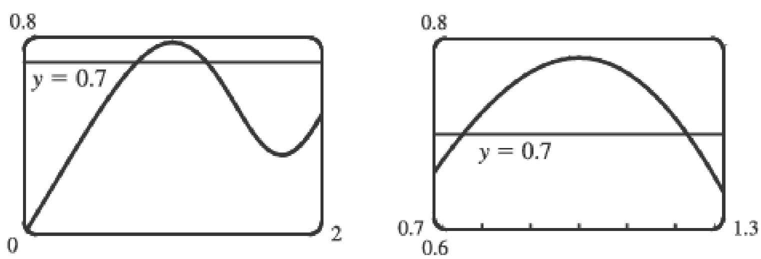
38. Since $\sqrt{x} - 1 < 0$ for $0 \le x < 1$ and $\sqrt{x} - 1 > 0$ for $1 < x \le 4$, we have $\left|\sqrt{x} - 1\right| = -\left(\sqrt{x} - 1\right) = 1 - \sqrt{x}$ for $0 \le x < 1$ and $\left|\sqrt{x} - 1\right| = \sqrt{x} - 1$ for $1 < x \le 4$. Thus,

$$\int_0^4 \left|\sqrt{x} - 1\right|\,dx = \int_0^1 \left(1 - \sqrt{x}\right)dx + \int_1^4 \left(\sqrt{x} - 1\right)dx = \left[x - \tfrac{2}{3}x^{3/2}\right]_0^1 + \left[\tfrac{2}{3}x^{3/2} - x\right]_1^4$$
$$= \left(1 - \tfrac{2}{3}\right) - 0 + \left(\tfrac{16}{3} - 4\right) - \left(\tfrac{2}{3} - 1\right) = \tfrac{1}{3} + \tfrac{16}{3} - 4 + \tfrac{1}{3} = 6 - 4 = 2$$

62. (a) C is increasing on those intervals where C' is positive. By the Fundamental Theorem of Calculus, $C'(x) = \dfrac{d}{dx}\left[\int_0^x \cos\left(\frac{\pi}{2}t^2\right)dt\right] = \cos\left(\frac{\pi}{2}x^2\right)$. This is positive when $\frac{\pi}{2}x^2$ is in the interval $\left(\left(2n - \frac{1}{2}\right)\pi, \left(2n + \frac{1}{2}\right)\pi\right)$, n any integer. This implies that $\left(2n - \frac{1}{2}\right)\pi < \frac{\pi}{2}x^2 < \left(2n + \frac{1}{2}\right)\pi \quad \Leftrightarrow \quad 0 \le |x| < 1$ or $\sqrt{4n-1} < |x| < \sqrt{4n+1}$, n any positive integer. So C is increasing on the intervals $(-1, 1)$, $\left(\sqrt{3}, \sqrt{5}\right)$, $\left(-\sqrt{5}, -\sqrt{3}\right)$, $\left(\sqrt{7}, 3\right)$, $\left(-3, -\sqrt{7}\right)$, ...

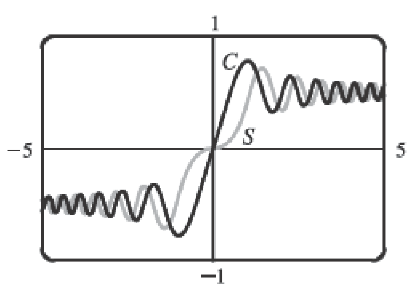
(b) C is concave upward on those intervals where $C'' > 0$. We differentiate C' to find C'': $C'(x) = \cos\left(\frac{\pi}{2}x^2\right) \quad \Rightarrow$ $C''(x) = -\sin\left(\frac{\pi}{2}x^2\right)\left(\frac{\pi}{2}\cdot 2x\right) = -\pi x \sin\left(\frac{\pi}{2}x^2\right)$. For $x > 0$, this is positive where $(2n-1)\pi < \frac{\pi}{2}x^2 < 2n\pi$, n any positive integer $\quad \Leftrightarrow \quad \sqrt{2(2n-1)} < x < 2\sqrt{n}$, n any positive integer. Since there is a factor of $-x$ in C'', the intervals of upward concavity for $x < 0$ are $\left(-\sqrt{2(2n+1)}, -2\sqrt{n}\right)$, n any nonnegative integer. That is, C is concave upward on $\left(-\sqrt{2}, 0\right)$, $\left(\sqrt{2}, 2\right)$, $\left(-\sqrt{6}, -2\right)$, $\left(\sqrt{6}, 2\sqrt{2}\right)$, ...

(c)



From the graphs, we can determine that $\int_0^x \cos\left(\frac{\pi}{2}t^2\right)dt = 0.7$ at $x \approx 0.76$ and $x \approx 1.22$.

(d)



The graphs of $S(x)$ and $C(x)$ have similar shapes, except that S's flattens out near the origin, while C's does not. Note that for $x > 0$, C is increasing where S is concave up, and C is decreasing where S is concave down. Similarly, S is increasing where C is concave down, and S is decreasing where C is concave up. For $x < 0$, these relationships are reversed; that is, C is increasing where S is concave down, and S is increasing where C is concave up. See Example 5.3.3 and Exercise 5.3.65 for a discussion of $S(x)$.

70. $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^9 + \left(\frac{2}{n}\right)^9 + \left(\frac{3}{n}\right)^9 + \cdots + \left(\frac{n}{n}\right)^9\right] = \lim_{n\to\infty} \frac{1-0}{n}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^9 = \int_0^1 x^9\,dx = \left[\frac{x^{10}}{10}\right]_0^1 = \frac{1}{10}$

The limit is based on Riemann sums using right endpoints and subintervals of equal length.