ANALYSIS AND OPTIMIZATION: MIDTERM 1 PRACTICE PROBLEMS

SPRING 2016

(1) Find the global minimum and maximum of the function $f(x) = x^3/3 + x^2/2 - 2x$ on the set $[-3,3]$.

(2) Calculate the rank of the matrix

$$\begin{pmatrix} 2 & 1 & 3 & 7 \\ -1 & 4 & 3 & 1 \\ 3 & 2 & 5 & 11. \end{pmatrix}$$

Are columns 1, 2, and 3 linearly independent?

(3) Give an example of the following or explain why an example does not exist.
  (a) A continuous function $f : [0,1] \to \mathbf{R}$ whose image is unbounded.
  (b) A continuous function $f : \mathbf{R} \to \mathbf{R}$ whose image is not closed.
  (c) A convex set without any extreme points.
  (d) A convex set with infinitely many extreme points.
  (e) A subset of $\mathbf{R}^2$ that is neither open nor closed.
  (f) A subset of $\mathbf{R}$ that is both open and closed.

(4) Suppose $S \subset \mathbf{R}^n$ is a convex set. Let A be an $m \times n$ matrix. Show that the set

$$T = \{\vec{y} \in \mathbf{R}^n \mid \vec{y} = A\vec{x} \text{ for some } \vec{x} \in S\}$$

is a convex set.

(5) Use Gaussian elimination to solve the equations

$$\begin{aligned} x + y + 2z &= 1 \\ -x + y + 3 &= 0 \\ y + 3z &= 2. \end{aligned}$$

(6) Find the determinant of

$$\begin{pmatrix} 2 & 3 & 1 & 0 \\ 4 & -2 & 0 & -3 \\ 8 & -1 & 2 & 1 \\ 1 & 0 & 3 & 4 \end{pmatrix}$$

(7) Let $S \subset \mathbf{R}^n$ be convex, and $\vec{x}, \vec{y}, \vec{z} \in S$. Let $a, b, c \in \mathbf{R}$ be such that $0 \le a, b, c \le 1$ and $a + b + c = 1$. Show that $a\vec{x} + b\vec{y} + c\vec{z} \in S$.

(8) Do the word problem from LEF 9.3, problem 25. How much extra profit is obtained by increasing the allowed assembly time by an hour? Increasing painting time by an hour? Increasing packaging time by an hour?

(9) Do the following maximization problems using the simplex method (the link to LEF is on the course webpage)
  (a) LEF 9.3, problem 14
  (b) LEF 9.3 problem 18

  For each of these problems, what is the shadow price associated with each constraint?

(10) Minimize $z = 14x + 20y$ subject to the constraints $x + 2y \geq 4$, $7x + 6y \geq 20$, and $x, y \geq 0$ by plotting the feasible set.
Formulate and solve the dual problem graphically.
If we change the constraint (in the primal) to $x + 2y \geq 4 + \epsilon$, what would be the change in the optimal z?

(11) Do the minimization problem: LEF 9.4, problem 8.

(12) State the maximum theorem.

(13) Do LEF 9.3 problem 10. Formulate the dual minimization problem. What is the optimum solution of the dual?

(14) Determine if the following sets are open or closed or neither. Justify your answers.
  (a) $S \subset \mathbf{R}^3$ defined by $S = \{\vec{x} \mid 2 < |\vec{x}| < 3\}$.
  (b) $S \subset \mathbf{R}^2$ defined by $S = \{(x, y) \mid x^2 + y > 3 \text{ and } x + y < 1\}$.
  (c) $S \subset \mathbf{R}$ defined by $S = \{x \mid x = 1/n + 1/m$ for some positive integers m and $n\}$.

(15) Let $S \subset \mathbf{R}^n$ be a set. Let S° be the set of interior points of S. Show that S° is an open set. What is S° for $S = [0, 1] \subset \mathbf{R}$? For $S = \{0, 1\} \subset \mathbf{R}$?