ANALYSIS AND OPTIMIZATION: HOMEWORK 8

SPRING 2016

Due date: Wednesday, April 20.

(1) Find the maximum value of
$$1-(x-1)^2-e^{y^2}$$
subject to the constraint
$$x^2+y^2 \leq 1.$$
Be sure to check constraint qualification. Explain why your solution is a global maximum.

(2) Given real numbers $\alpha > 0$ and $\beta > 0$ with $\alpha+\beta \leq 1$, find the maximum value of $x^\alpha y^\beta$ subject to
$$x+y \leq 2, \quad x \geq 0, \quad y \geq 0.$$
Verify that your answer satisfies the envelope theorem.

(3) Minimize x^2+y^2 subject to
$$y^2 \leq (x-1)^3.$$
Does a Lagrange multiplier exist? Explain what happened.

(4) A manufacturer produces three types of plastic fixtures. The time required for molding, trimming, and packaging are given in the following table (in hours per fixture).

Process	Type A	Type B	Type C	Total time available
Molding	1	2	3/2	120
Trimming	2/3	2/3	1	46
Packaging	1/2	1/3	1/2	24
Profit	\$11	\$16	\$15	–

Consider the solution $(A = 6, B = 51, C = 8)$. Show that the KKT conditions are satisfied at this point, and find the Lagrange multipliers. Show that this point is a global maximum. How much would the profit increase if (a) the trimming time for type A was decreased by ϵ, (b) the packaging time for type C was decreased by ϵ?

(5) For a given real number c, find the maximum value of $f(x,y) = cx+y$ on the set defined by
$$x^2+2y^2 = 2, \quad x \geq 0 \quad y \geq 0.$$
Sketch the value function $f^*(c)$.