Excellent 60

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Analysis/Optimiz.

HW 1

1) $f(x) = \frac{x^3}{3} + \frac{x^2}{2} - 2x$

$f'(x) = x^2 + x - 2$

Setting $f'(x)=0$ gives critical points:

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$ are the critical points

$f''(x) = 2x + 1$

Evaluate $f''(x)$ at critical points:

$f''(-2) = -3$ So $x=-2$ is the local max

$f''(1) = 3$ So $x=1$ is the local min

Evaluate $f(x)$ at the critical points and boundaries:

$f(-2) = 10/3$

$f(1) = -7/6$ So over the interval, $x=1$ is the global minimum and $x=3$ is the global maximum.

$f(-3) = 3/2$

$f(3) = 15/2$

If we take $f(x)$ to be defined over the real line, we get a global min at $x=-\infty$ and a global max at $x=\infty$ (there is no global min/max).

2) $f(x) = x^x$

First rewrite x^x as a general exponential:

$f(x) = x^x = e^{\ln(x)\cdot x}$

Now set $u = \ln(x)\cdot x$ and differentiate:

$f'(x) = d/dx\left(x\cdot\ln(x)\right)\cdot e^{\ln(x)\cdot x}$

$= (\ln(x)+1)e^{x\ln(x)}$

$= x^x(1+\ln(x))$

This is the final derivative.

Check for critical points:

$f'(x) = 0$

$x^x(1+\ln(x)) = 0$

$x^x = 0$ or $1+\ln(x) = 0$

no sols. $\ln(x) = -1$

$x = e^{-1}$

Evaluate critical points and boundaries:

$f(0) = 0^0$ = undefined

$\lim_{x \to 0} x^x = 1$

$f(\infty) = \infty$

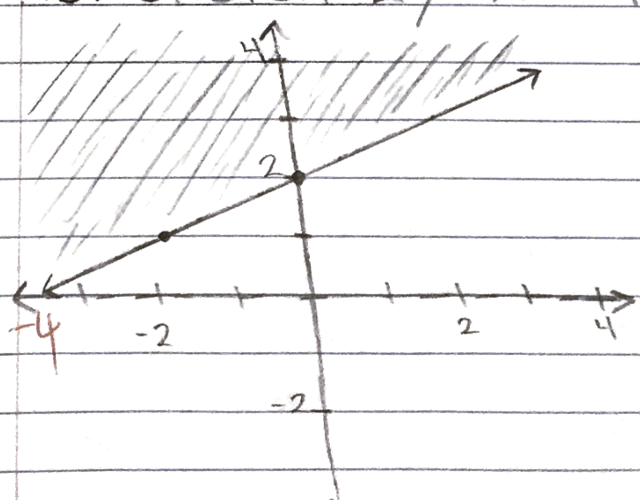
$f(e^{-1}) = e^{-1/e} \approx .6922$ So global minimum on $[0, \infty)$ is at $x = e^{-1}$.

3)

1. F
2. C
3. A
4. D
5. B
6. E

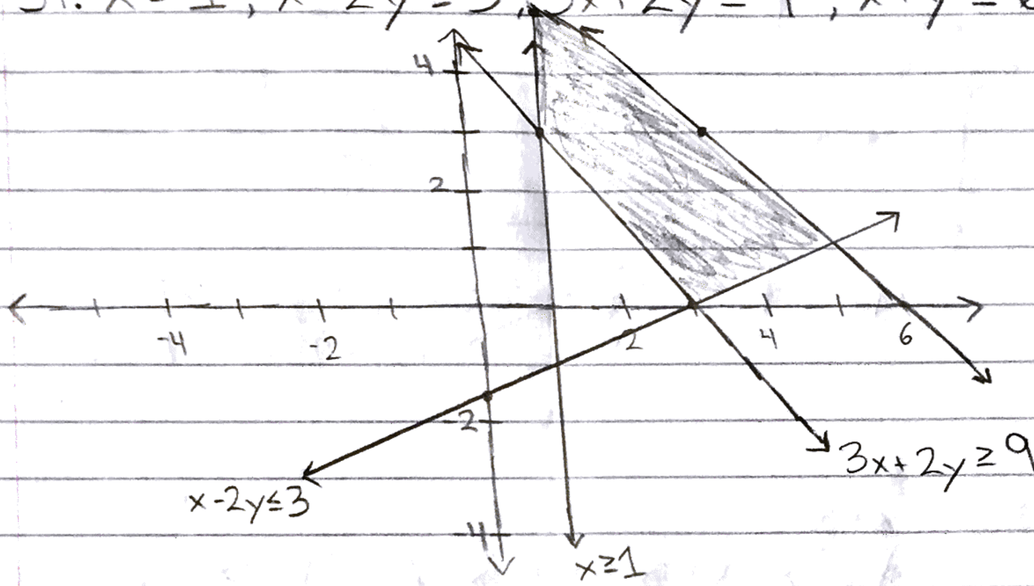
4)

13. sketch $2y - x \geq 4$



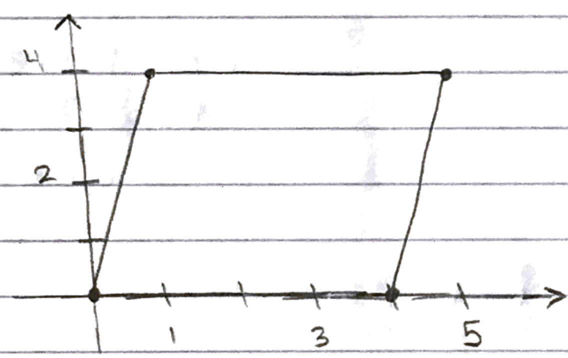
5)

31. $x \geq 1,\ x - 2y \leq 3,\ 3x + 2y \geq 9,\ x + y \leq 6$



6)

34.



This could be described by:

$y \leq 4$

$y \geq 0$

$y \leq 4x$

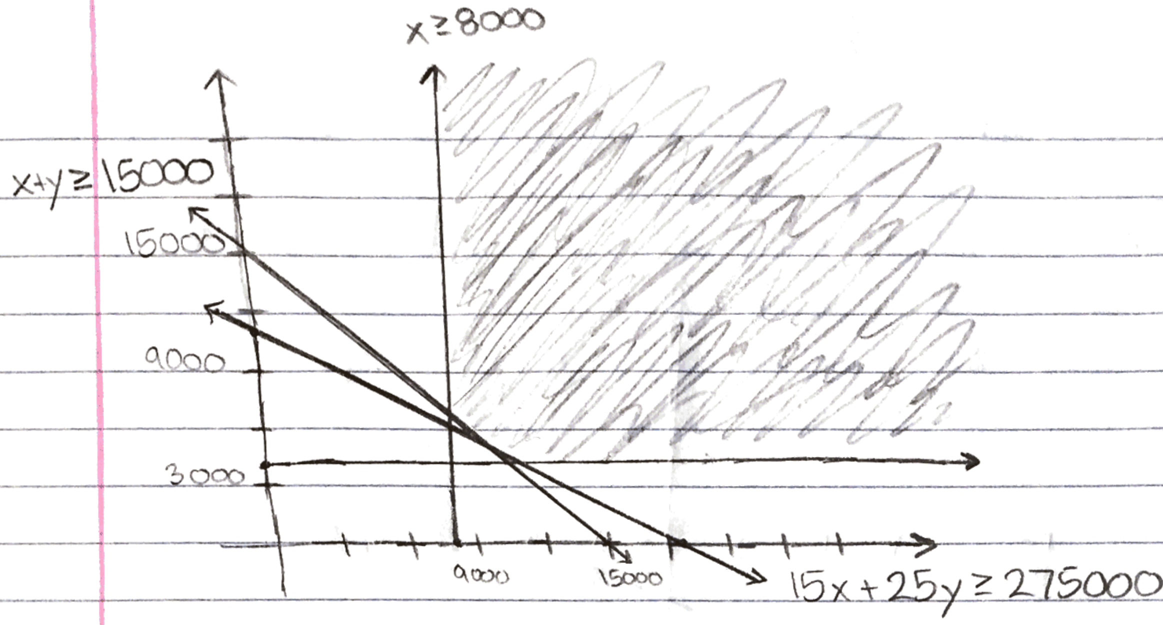
$y \geq 4x - 16$

7)

40. let $x =$ # of \$15 tics, $y =$ # of \$25 tics

$x \geq 8000$, $y \geq 4000$

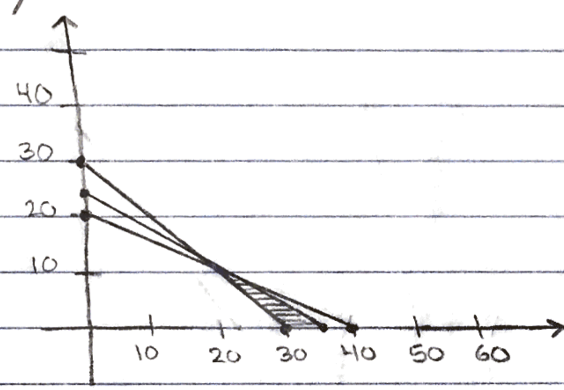
$x + y \geq 15000$ and $15x + 25y \geq 275000$



8)

13. $z = 4x + y$

subject to $x \geq 0$, $y \geq 0$, $x + 2y \leq 40$, $x + y \geq 30$, $2x + 3y \geq 72$



We have vertices $(36,0)$, $(40,0)$ and $(24,8)$

Test each vertex:

$(36,0) \longrightarrow z = 144$

$(40,0) \longrightarrow z = 160$ maximum of 160 at $(40,0)$

$(24,8) \longrightarrow z = 104$ minimum of 104 at $(24,8)$

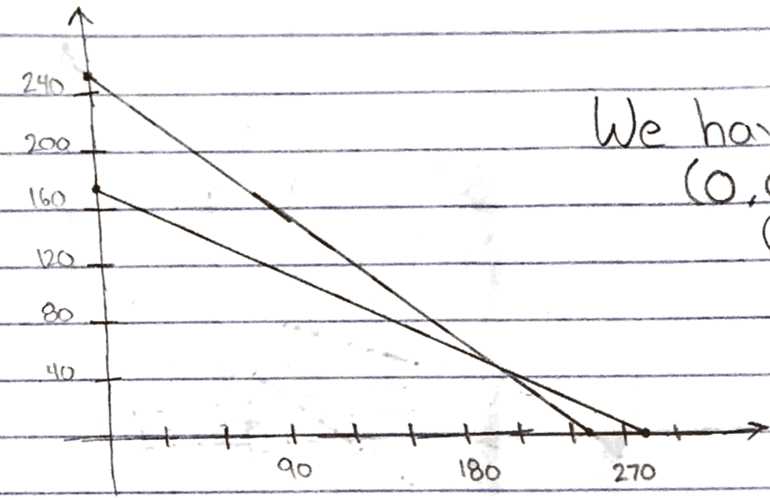
9)

21. let x = # of \$250 com., y = # of \$400 com.

$x \geq 0$, $y \geq 0$, $x + y \leq 250$

$250x + 400y \leq 70000$

with objective function $z = 45x + 50y$



We have vertices (0,0), (250,0), (0,175) and (200,50).

Test each vertex:

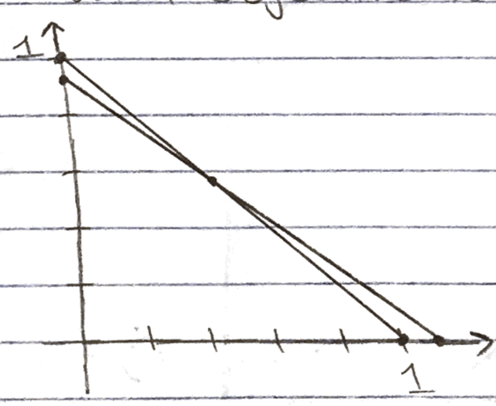
(0,0) → profit = 0
(250,0) → profit = 11250
(0,175) → profit = 8750
(200,50) → profit = 11500

Maximum profit of \$11500 by stocking 200 \$250 machines and 50 \$400 machines.

10)

24. let x = fraction type A, y = fraction type B
We assume our product is all type A or type B.

$x \geq 0,\ y \geq 0,\ x + y = 1,\ .8x + .92y \geq .9$
with objective function $z = .83x + .98y$



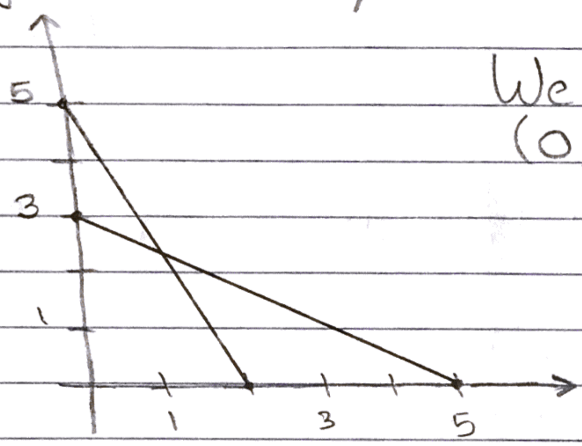
$.8x + .92y = .9,\ y = 1 - x$
$.8x + .92(1-x) = .9$
$-.12x = -.02$
$x = 1/6$
$y = 1 - x = 5/6$
$z = .83(1/6) + .98(5/6) = .955$

Minimum cost of \$.955 per liter achieved with blend of 1/6 type A and 5/6 type B.

11)

25. $z = 2.5x + y$

subject to $x \geq 0, y \geq 0, 3x + 5y \leq 15, 5x + 2y \leq 10$



We have vertices $(0,0), (2,0), (0,3)$ and $(\frac{20}{19}, \frac{45}{19})$.

Test each vertex:

$(0,0) \rightarrow z = 0$

$(2,0) \rightarrow z = 5$

$(0,3) \rightarrow z = 3$

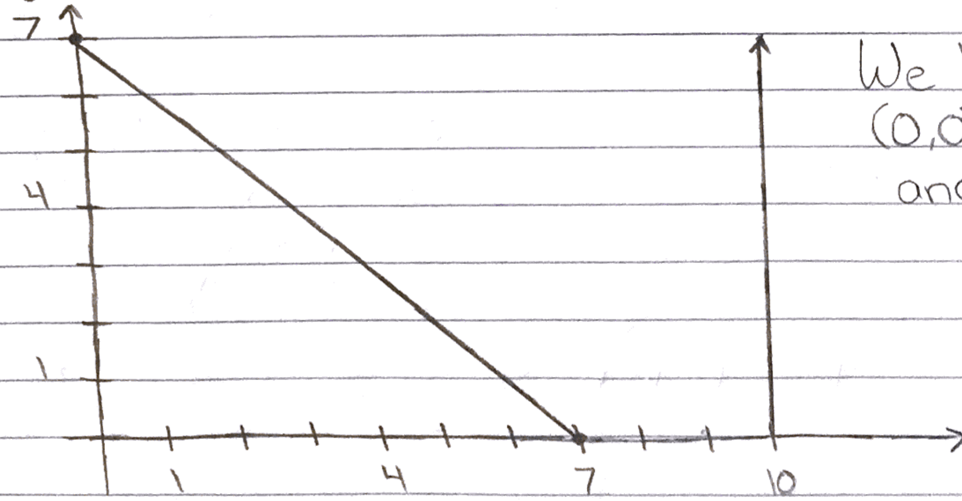
$(\frac{20}{19}, \frac{45}{19}) \rightarrow z = 5$

The unusual characteristic is that two vertices both give the same maximum value (objective is maximized with $z = 5$ at $(2,0)$ and $(\frac{20}{19}, \frac{45}{19})$).

12)

27. $z = -x + 2y$

subject to $x \geq 0, y \geq 0, x \leq 10, x + y \leq 7$



We have vertices $(0,0), (7,0)$, and $(0,7)$.

Test each vertex:

$(0,0) \rightarrow z = 0$

$(7,0) \rightarrow z = -7$

$(0,7) \rightarrow z = 14$ Maximum of 14 at $(0,7)$

The unusual characteristic is that we have a constraint that does not actually affect our problem ($x \leq 10$ is redundant since $x + y \leq 7$ and $y \geq 0$ already imply that $x \leq 7$).