MODERN ALGEBRA 2: HOMEWORK 4

(1) Chapter 12, § 2.1

(2) Chapter 12, § 2.4

(3) Chapter 12, § 2.6

(4) Chapter 12, § 5.1 (Do any two)

(5) Chapter 12, § 5.3

(6) Chapter 12, § 5.6

(7) Chapter 12, § 5.7

(8) Show that the equation $x^2 - 2y^2 = 5$ has no solutions where $x, y \in \mathbf{Z}$.

(9) Show that the equation $x^2 - 2y^2 = 7$ has infinitely solutions with $x, y \in \mathbf{Z}$.

MORE FOOD FOR THOUGHT

If you have time and enjoy thinking about these kinds of arithmetic questions, think about the following. You don't have to write up the solutions and turn them in.

(1) Can you find *all* the integer solutions of $x^2 - 2y^2 = 7$? What would you like to know about $\mathbf{Z}[\sqrt{2}]$ to answer this?
(2) We proved in class that a positive odd prime integer can be written as a sum of two squares if and only if it is congruent to $1 \pmod 4$. Can you complete this to a characterization of the positive integers (not necessarily prime) that can be written as sums of two squares?
(3) The ring of Eisenstein integers has something to do with writing a number as $a^2 + ab + b^2$. See if you can figure out a precise statement.