MODERN ALGEBRA 1: HOMEWORK 7

(1) Find an n for which $\mathbf{Z}_n^\times$ is cyclic and one n for which it is not.

Caution. $\mathbf{Z}_n^\times$ is *not* a subgroup of $\mathbf{Z}_n^+$. The operations are different.

(2) Draw two plane figures each of which has exactly 8 symmetries but such that their symmetry groups are not isomorphic.

(3) Chapter 6: 4.1

(4) Chapter 6: 4.3

(5) Chapter 6: 6.3 (You don't need to write justifications, but convince yourself, or better, your friend that your answer is right.)

(6) Let n be a positive integer. Define O_n and SO_n by

$$O_n = \text{Set of } n \times n \text{ matrices } M \text{ satisfying } M^T M = I$$
$$SO_n = \text{Set of } n \times n \text{ matrices } M \text{ satisfying } M^T M = I \text{ and } \det M = 1.$$

Show that O_n is a subgroup of $\mathrm{GL}_n(\mathbf{R})$ and SO_n is a normal subgroup of O_n. What is the quotient O_n/SO_n?

(7) (a) Show that SO_2 is just the group of rotation matrices

$$SO_2 = \left\{ \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \mid \theta \in \mathbf{R} \right\}.$$

(b) Show that O_2 is given by

$$O_2 = SO_2 \cup \left\{ \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \mid \theta \in \mathbf{R} \right\}.$$

What transformation does the matrix $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ describe?

(c) It true that $O_2 \cong SO_2 \times \{\pm 1\}$?