MODERN ALGEBRA 1: HOMEWORK 10

(1) Chapter 7, 2.5

(2) Chapter 7, 2.9 (a), (b).

(3) Chapter 7, 2.14

(4) Show that $\mathrm{Aut}(\mathbf{Z}/n\mathbf{Z}) \cong (\mathbf{Z}/n\mathbf{Z})^\times$.

(5) Let p be a prime. Show that a group order p^2 is either isomorphic to $\mathbf{Z}_{p^2}$ or $\mathbf{Z}_p \times \mathbf{Z}_p$. (Recall that we know it must be abelian.)

(6) Show that there are at least two (non-isomorphic) groups of order 21.

(7) We have a G with $N \triangleleft G$ and a complementary H. Suppose H is also a normal subgroup of G. Show that $G \cong N \times H$ (direct product).

(8) Find three groups G with a normal subgroup $N \triangleleft G$ where $N \cong \mathbf{Z}$ and $G/N \cong \mathbf{Z}_2$. Be sure to explain why no two of your examples are isomorphic to each other.