Practice Exam 2

Modern Algebra I, Dave Bayer, March 24, 2009

Name: ______________________________

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[**1**] Prove that if a prime p divides the order of a finite group G, then G contains an element of order p.

[2] Let G be a finite group of order p^e where p is prime. Define the *center* Z of G, and prove that Z has order greater than 1.

[3] Let A_5 be the alternating group of even permutations on five elements. Define a *simple* group. Prove that A_5 is simple.

[**4**] Write the order of a finite group G as $n = p^e m$ where p is prime and p does not divide m. Prove that there is a subgroup of G whose order is p^e.

[5] Let G be a finite group acting on a finite set X. Prove *Burnside's formula:* The number of orbits of this action is

$$\frac{1}{|G|}\sum_{g\in G}|X^g|$$

where X^g denotes the set of elements in X that are fixed by g.