What is alg. geo.?

$$X^n + Y^n = Z^n$$ has no non-const. sol^{n}s. in $\mathbb{F}_p, k[t]$, $\mathbb{C}[t]$, $\mathbb{R}$

$n \geq 3$

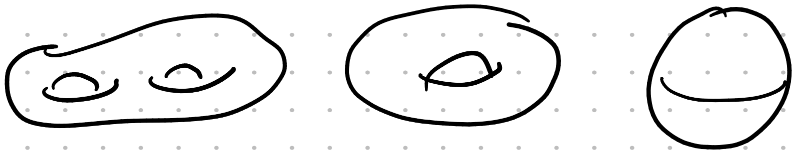
$\mathbb{C}$-solutions of $X^n + Y^n = Z^n \subset \boxed{\mathbb{C}^3 \setminus 0 / \text{scaling}}$

(x, y, z) sol^n $\Rightarrow$ $(\lambda x, \lambda y, \lambda z)$ $\quad \lambda \in \mathbb{C}$

compact
smooth
of $\mathbb{C}$-dim 2 $\leftarrow$ $\mathbb{P}^2 = (\mathbb{C}^3 \setminus \{0,0,0\}) / \text{scaling}$.

$\cup$

$V = \{X^n + Y^n = Z^n\}$ $\quad$ $\mathbb{C}$ dim 1 compact $\quad$ (2 top dim) orientable



up to Homeo

$\boxed{g = \binom{n-1}{2}}$

Thm

$2-2g$

g "genus"

$n=1 \rightsquigarrow 0$

$n=2 \rightsquigarrow 0$

$n=3, \ldots \rightsquigarrow >0$

Suppose $F, G, H \in \mathbb{C}[t]$ was a sol^n.

$\forall\ t \in \mathbb{C} \rightsquigarrow [F(t) : G(t) : H(t)] \in V$

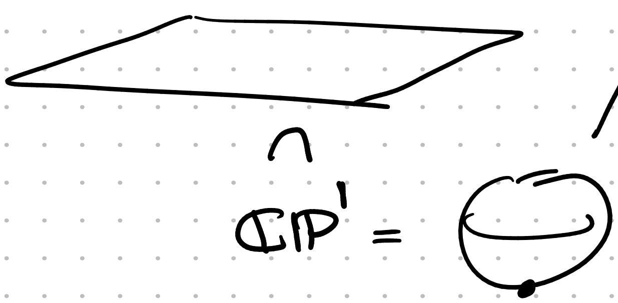
$\mathbb{C} \longrightarrow V$

(non-const if we have non const)

$[t^3 : 2t^3 + 1 : t^2 - 15]$

$\wr$

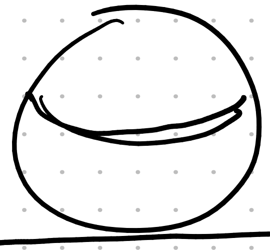
$[1 : 2 : 0]$



$\cap$

$\mathbb{CP}^1 =$

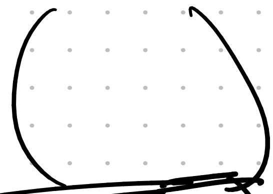
2.

 $\xrightarrow{\pi}$ V genus 0 or negative.

Claim: A non-const map like this is only possible if $g(V) = 0$.

Pf:- π is a covering space over the compl. of finitely many pts. $\{P_1 - - -\}$

fewer than d.

π^{-1}  $\xrightarrow{d:1}$ $\underbrace{V \setminus \{P_1, \dots, P_n\}}$

$\chi = d\ \chi(\underline{\quad})$

& with each punct. goes down by 1

$$\boxed{\begin{aligned} \underbrace{\chi(\ominus)}_{2} &= |\pi^{-1}\{P_1, \dots, P_n\}| \\ &\quad + d\,(\chi(V \setminus \{P_1, \dots, P_n\})) \\ &= \underbrace{d\,\chi(V)}_{\leq 0} + \underbrace{|\pi^{-1}(\ )| - d\,|\{P_1, \dots, P_n\}|}_{\leq 0} \end{aligned}}$$